Physics

Teachers' guide **Unit 4**

Waves and oscillations

Nuffield Advanced Science

PHYSICS DEPT.
MORAY HOUSE
College of Education
EDINBURGH

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Waves and oscillations

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Nuffield Advanced Science

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Waves and oscillations

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Foreword

It is almost a decade since the Trustees of the Nuffield Foundation decided to sponsor curriculum development programmes in science. Over the past few years a succession of materials and aids appropriate to teaching and learning over a wide variety of age and ability ranges has been published. We hope that they may have made a small contribution to the renewal of the science curriculum which is currently so evident in the schools.

The strength of the development has unquestionably lain in the most valuable part that has been played in the work by practising teachers and the guidance and help that have been received from the consultative committees to each Project.

The stage has now been reached for the publication of materials suitable for Advanced courses in the sciences. In many ways the task has been a more difficult one to accomplish. The sixth form has received more than its fair share of study in recent years and there is now an increasing acceptance that an attempt should be made to preserve breadth in studies in the 16–19 year age range. This is no easy task in a system which by virtue of its pattern of tertiary education requires standards for the sixth form which in many other countries might well be found in first year university courses.

Advanced courses are therefore at once both a difficult and an interesting venture. They have been designed to be of value to teacher and student, be they in sixth forms or other forms of education in a similar age range. Furthermore, it is expected that teachers in universities, polytechnics, and colleges of education may find some of the ideas of value in their own work.

If the Advanced Physics course meets with the success and appreciation I believe it deserves, it will be in no small measure due to a very large number of people, in the team so ably led by Jon Ogborn and Dr Paul Black, in the consultative committee, and in the schools in which trials have been held. The programme could not have been brought to a successful conclusion without their help and that of the examination boards, local authorities, the universities, and the professional associations of science teachers.

Finally, the Project materials could not have reached successful publication without the expert assistance that has been received from William Anderson and his editorial staff in the Nuffield Science Publications Unit and from the editorial and production teams of Penguin Education.

K. W. Keohane
Co-ordinator of the Nuffield Foundation Science Teaching Project

The Teachers' guide

This volume is intended to contain whatever information and ideas are required for the day to day teaching of the Unit. Not every teacher will need all of it all of the time: sometimes the summary and the list of experiments will come nearer to meeting the need.

The main text contains, on the righthand pages, a detailed suggested teaching sequence, which teachers can adopt or adapt. The facing lefthand pages carry practical details, suggested questions, references, and background information for teachers in the form of a commentary on the text. This commentary also indicates aims of the teaching, and points out links with other parts of the course.

At the end, there are some appendices containing material needed on occasion only, and lists of apparatus and teaching aids for the Unit. These include details of books and articles referred to in this *Guide*.

Introduction

Unit 4 is about waves and oscillations. In the form and order suggested, it is intended to suit students who have been through the work on waves in the Nuffield O-level Physics course, or equivalent work in other courses. In particular, the ripple tank studies suggested in Nuffield O-level Physics, *Teachers' guide III,* Chapter 1, and *Guide to experiments III,* experiment 4, are essential background; so is a first acquaintance with the wave properties of light.

With such students in mind, Part One begins with invisible waves, and develops, out of experiments on superposition and on the speed of the waves, some evidence for the existence of the electromagnetic spectrum.

Part Two considers mechanical waves, showing in detail how the speed of one example of such waves can be understood from first principles. This is in part a preparation for Unit 8, '*Electromagnetic waves*', in which it will be necessary for students to see that it may be possible to do the same for the speed of radio waves.

Part Three continues the movement from the general to the particular, and considers simple harmonic motion in some detail. It concludes with a discussion of standing waves, intended to draw together the threads developed so far, linking oscillations, superposition, and resonance. This discussion also prepares for Unit 10, *Waves, particles, and atoms,* where standing wave ideas are needed.

Part One is almost wholly empirical, and is also very general in the sweep of the ideas considered. This is deliberate, as a break from the rather detailed and theoretical arguments about electric charge, field, and potential that precede it in Unit 3, *Field and potential.* It is also intended to make use of students' previous experience of visible waves.

Teachers whose students have little prior experience of waves may do better to begin with some individual experimenting with ripple tanks, and then to start Unit 4 at Part Two. Part One could follow Part Two, or could come last of all.

This Unit is intended to contribute to a number of the general aims of the course. It includes some small but open investigations (4.1 and 4.13) in which the student's main task is to think what to do for himself or herself. On the other hand, it contains one or two long, detailed experiments (4.3, 4.4) which may help to develop patience, persistence, and careful experimenting.

Part Three forms a significant part of the teaching of mathematics within the physics course. The motion of a harmonic oscillator is used to show the meaning and use of a second derivative (first derivatives are discussed in Unit 2 and again in Unit 5). Numerical–graphical methods are again suggested as a fruitful teaching device. They will be used again in Unit 10. At the same time, the work on simple harmonic motion is used to illuminate the uses and limitations of mathematical models within physics.

The time at which Unit 4 can be taken within the course is not very tightly constrained. If it is taken before Unit 3, the connections that are made with Unit 3 and with Unit 1 can be transferred to the teaching of these Units. For example, the resonant frequency of ions in sodium chloride (page 93) could appear in the teaching of Unit 3, Part Four ('Ionic crystals'). But Unit 4 cannot easily come much later than is suggested.

Summary of Unit 4

Time: four to five weeks.

(Numbers in brackets refer to suggested experiments, listed on page 6.)

Part One

Waves of many sorts

Time: about a week.

This Part looks at some of the purely experimental evidence for the existence of the electromagnetic wave family. It considers superposition effects and evidence from the wave speed.

Suggested sequence

Superposition effects with radio waves, microwaves, light, and sound (4.1), treated as student investigations. The electromagnetic spectrum, including infra-red and ultra-violet radiation (4.2). The speed of light (4.3) and of microwaves (4.4).

Part Two

Mechanical waves

Time: about a week.

This Part looks first at superposition of mechanical wave pulses. Then it discusses the speed of one type of mechanical wave, following this with a review of a wider variety of such waves.

Suggested sequence

Superposition of wave pulses on springs, and on mechanical wave models (4.5). Understanding the speed of compression waves in a solid (4.6, 4.7), as an example of the possibility of explaining a wave speed from basic principles. Review of other mechanical waves (4.8), giving an opportunity for much background reading.

Part Three

Mechanical oscillations

Time: two to three weeks.

This Part mainly develops a mathematical model for discussing simple harmonic motion, having first identified such motion as a special and simple type of oscillation. It illustrates the meaning of a second derivative, and the solution of a second order differential equation. Finally, the threads are drawn together in a discussion of resonance and standing waves.

Suggested sequence

Repetitive events and the idea of time (4.9), followed by the identification of simple harmonic motion (4.10). Detailed discussion of simple harmonic motion (4.11, 4.12) leads to the construction of a mathematical model, and to $T = \frac{2\pi}{\sqrt{k/m}}$.
Resonance (4.13, 4.14), using both student investigation and demonstration.
Standing waves (4.15, 4.16) on strings and in more than one dimension.

Choosing one's own path

We hope and expect that teachers will find their own ways of using the material in this Unit. The detailed teaching programme laid out in the following pages represents as good a way of handling the material as we have been able to find in the light of experience in the trials, but should not be thought of as more than a possible, fairly well tested way of achieving the aims we decided upon. No doubt others can and will do better.

But teachers will know that it is the detail that counts in successful teaching, and so the *Guide* is full of particular teaching suggestions and practical details. We hope that these will help those who are uncertain how to handle either new material, or old material taught in a new way for unfamiliar aims.

The summary and list of experiments will, it is hoped, assist those who have taught the course a few times and no longer need to refer to all of the detailed teaching suggestions, as well as those who feel confident that they can make up their own teaching programme out of their previous experience. We also hope that the summary will give all teachers an overall view of the work suggested. Such a view is necessary for keeping a sense of perspective and direction, both when one is immersed in particular detailed teaching suggestions and comments, and when students lead the teaching off in an unpredictable direction by contributing their own ideas.

It seems fair to add that the summary, taken on its own, could mislead. It cannot easily indicate the aims of pieces of work in any precise way, or find words to express the relative seriousness or lightness of particular episodes. Nor should a phrase one might also find in a current examination syllabus always be taken here to imply the same work as it would imply there.

Experiments suggested for Unit 4

4.1 Superposition of waves *page 11*

a Superposition of 1 GHz radio waves *page 15*

b Superposition of microwaves *page 19*

c Superposition effects with v.h.f. radio transmissions or u.h.f. television transmissions *page 21*

d Superposition of light waves *page 23*

e Superposition of sound waves *page 25*

4.2 Infra-red and ultra-violet radiation *page 29*

4.3 Measurement of the speed of light *page 31*

4.4 Measurement of the speed of microwaves *page 33*

4.5 Mechanical waves *page 39*

a Transverse waves on a long narrow spring *page 41*

b Longitudinal and transverse waves on a Slinky spring *page 43*

c Transverse waves on a wave model made of trolleys and springs *page 43*

d Waves on water *page 45*

4.6 Longitudinal wave on a wave model made of trolleys and springs *page 47*

4.7 Speed of sound in a metal rod *page 53*

4.8 Testing other wave speed expressions *page 57*

4.9 What is a clock? *page 61*

4.10 The motion of oscillators *page 67*

4.11 What factors determine the period of an oscillator? *page 71*

4.12 Detailed study of the motion of one oscillator *page 73*

4.13 Resonance in a simple system *page 97*

4.14 Barton's pendulums *page 99*

4.15 Standing waves on springs and strings *page 103*

4.16 More complicated standing waves *page 107*

Part One

Waves of many sorts

Time: about a week

General, empirical survey of electromagnetic waves

Part One takes a very general point of view, with experiments 4.1 to 4.4 all being intended to suggest how the observed properties of radio waves, light, and infra-red and ultra-violet radiation may, or may not, seem to accord with the view that all belong to the same family. All have wave properties, and all travel at the same speed. These characteristics, together with the vaguer evidence from the similarities between the means of producing and detecting radiations close in wavelength but different in name, as well as similarities between some of their effects, suggest, without clinching the point, that it is reasonable to speak of one electromagnetic spectrum.

As pointed out in the Introduction, this general point of view depends on earlier experience and knowledge, such as that offered by the Nuffield O-level Physics course. Students with a different background may need to start elsewhere. Given this background, the experiments grouped in 4.1 are mainly revision of the idea of wave superposition and the use of $v = f\lambda$, but in the context of invisible waves rather than the visible water waves so much used at O-level.

The series of experiments illustrates an interesting point about the way physics works. With waves one can see, like water waves, one can arrange that waves in step or out of step superpose, and look to see if the expected superposition (interference) effects occur. With waves one cannot see, the argument is reversed. The radiation may show such effects, and these may fit in with a calculated wavelength and some path differences. So one argues that the radiation may well be a wave. Physicists often find themselves arguing from what happens on the bench (meter readings, say) to the unseen events that might lie behind the observed events. This point can be made from time to time as the experiments progress.

The group of experiments in 4.1 has one more aim. As part of becoming better at devising an experiment for a purpose, instead of being asked to follow detailed instructions, students can be asked to think of things to do for themselves.

Reading

For uses of radio waves, see:
Battan, *Radar observes the weather.*
McLean, 'Colour television' (reprint).

Note For details of the above and other reading recommended in this *Guide*, see the list entitled 'Books and further reading' on page 139.

Group of experiments
4.1 Superposition of waves

Details of apparatus appear below, under 4.1a to e.

The apparatus for 4.1a, a 1 GHz oscillator with transmitting and receiving dipoles, is unfamiliar to students, and is the core of the group of experiments. It will help to show the apparatus working as a demonstration, so reducing the need for instructions about setting it up. The demonstration can be used to bring out the general purpose of all the experiments, and to illustrate how superposition effects could be made to occur. The wavelength is about 0.3 m, so the receiver readily indicates rises and falls of received radiation when some reaches it after reflection from a hand moved nearby.

The electromagnetic wave family

Unit 3 discussed electric fields; Unit 4 begins with electrical waves. These waves may include those carrying radio or television transmissions; they will include some made in the laboratory and transmitted and received by aerials like those on roof tops. They will also include microwaves, already seen in Unit 1, *Materials and structure.*

Light is also thought to belong to the same family as radio waves. The theory linking both with waves involving electric and magnetic fields is explored later on in the course (Unit 8, *Electromagnetic waves*). Just now, the behaviour of these waves is looked at experimentally, to observe the similarities and differences between the members of the family.

For contrast, sound waves are looked at too, as an example of a radiation that is a wave but is not a member of the electromagnetic wave family.

Radio waves have obvious practical importance. Besides radio and television, they are used for radar, radio navigation, and long range telephone links. They can be used to detect thunderstorms or to operate spacecraft by remote control. Radio astronomers detect radio waves from distant galaxies and can use the signals they receive to find out more about the Universe.

Group of experiments

4.1 Superposition of waves

The central question behind all the experiments in this group is the same: given some radiation, how can one tell whether or not it has wave properties, if the waves themselves are invisible? Experiments like a two-slit experiment with light offer an answer: if two (or more) trains of waves arrive at one place their oscillations may or may not be in step, with observable consequences. Further questions can bring out the important points, the more easily if the 1 GHz radio wave apparatus for experiment 4.1a is working on the bench.

The experiments suggested use:

4.1a	1 GHz radio waves	4 sets of apparatus
4.1b	microwaves	1 set of apparatus
4.1c	v.h.f. radio or u.h.f. television transmissions (optional)	
4.1d	light	several sets of apparatus can be provided
4.1e	sound	1 or 2 sets of apparatus

Teachers should choose the mixture of these that suits them best. At one extreme, if 4 sets of 4.1a are available, all students could look at 4.1a only, in large groups. The size of such groups can be reduced by encouraging some students to try the other experiments. Experiments with v.h.f. radio or u.h.f. television out of the laboratory may appeal to some. 4.1d with light is useful for recalling O-level work, or, as suggested below, a simpler two-slit experiment from O-level can be substituted if it has not been seen before. 4.1e, using sound, is important to point out the difference between waves which are and waves which are not electromagnetic, but it will be enough for one group of students to try this and report to the rest.

We advise a time allowance of at least two long practical sessions, so that each student may handle the 1 GHz apparatus, and try one other experiment. If necessary, more time should be given, so that students do have time to think about what to do, to try it, think again, and so on.

Superposition

The use of words suggested opposite is supposed to be helpful, not obligatory. Nor should 'interference' and 'diffraction' be left out of students' vocabulary, for they will meet them in books, in other reading, and in examination questions, and should understand them well enough for those purposes.

Students' book

Questions 1 to 5 revise ideas about superposition, path difference, and wavelength measurement. Questions 6 and 7 are about wavelength, frequency, and wave speed.

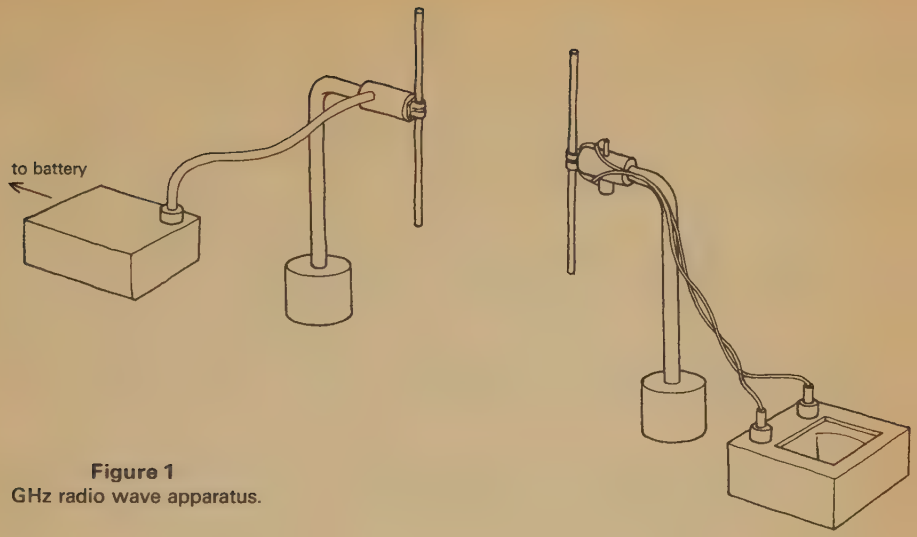


Figure 1
1 GHz radio wave apparatus.

Superposition

In a water wave the water surface moves up and down. In a sound wave the air expands and contracts. In both, the oscillations are around a mean value. In many cases, when two or more waves pass one place, the oscillation at that place is the sum of the oscillations of each wave (though not for two smooth humps of water in a shallow sea if the combined hump is big enough to make a breaker). The adding of oscillations must take account of sign, for the (positive) displacement of a hump above the mean will add to the (negative) displacement of a trough below the mean to produce less than either.

The effect is often called *interference,* though this may wrongly suggest only the cancelling-out effect, or that the waves permanently modify one another. Later on, the course will discuss *diffraction,* also a superposition effect, but one arising from the adding up of bits of wave with continuously varying phase, such as those that get through a hole, rather than the adding up of waves from a couple of sources or of two waves from one source that have a finite phase difference. The distinction is not very important, nor are names very consistent, for the 'diffraction' grating involves both. 'Superposition' means what it says and includes all these instances, as well as others like the Bragg 'reflection' of X-rays. It is a helpful word when one is thinking generally about waves, though one needs also to recognize the other two when they appear in books and articles.

Knowledge of frequencies

Sound is the only radiation whose frequency can be conveniently measured in a school laboratory, using a scaler and a stopwatch, or an oscilloscope.

That of the 1 GHz radiation is stamped on the box (the value is not accurate and varies from oscillator to oscillator) and could be measured with an oscilloscope, but this needs a better instrument than a school could possibly afford. It seems fair to take the marked value as a rough guide at least.

The frequency of radio and television transmissions is measured and guaranteed by the transmitting station, which has to keep it within limits. The frequency is published, and it seems reasonable to accept the values given. See Appendix A.

The frequency of the microwaves would be very hard to measure directly, if it were possible at all, and that of the light is impossible to measure. These are the two radiations for which an attempt is made to measure the speed directly, in experiments 4.3 and 4.4.

Experiment 4.1a Superposition of 1 GHz radio waves

1050	15 cm dipoles and oscillator (1 GHz)
503	retort stand base *2*
1002	microammeter
	or
181	general purpose amplifier
	and
183	loudspeaker (if not with above)
	or
1001	galvanometer (internal light beam)
	or
158	class oscilloscope
1053	metal screen *2*
1000	leads

1 GHz is a convenient frequency, because the wavelength is long enough for the aerials to look like familiar television aerials, but short enough for experiments to fit within limited laboratory space. Item 1050 is reliable and fairly cheap, but different equipment could be equally suitable.

The transmitting dipole is fed from the oscillator via a coaxial cable joining coaxial sockets on each. The dipole may also have a pair of 4 mm sockets, not used in this experiment, provided so that it can be operated as a spark transmitter in an experiment in Unit 8, *Electromagnetic waves*. The oscillator may require power from an external dry battery.

The receiving dipole may be connected to one of several detectors. An internal light beam galvanometer (1001) will enable radiation to be detected at about two metres, perhaps more, used on its most sensitive range. But at most only two groups can have this instrument.

The microammeter (1002) is less sensitive, detecting radiation at up to a metre. All groups can be supplied with this detector. But when the deflection is substantial, most of the effect is due to induction, not to electromagnetic radiation, the dipoles being only a few wavelengths apart. This detector is useful if space is short, as the transmissions of one group are less likely to be picked up by the receiver of another.

Wavelength, frequency, and wave speed

In each experiment the wavelength λ can be measured using a suitable path difference. If necessary, the process can be illustrated in a demonstration of 4.1a. If the frequency f is known, as it is for the 1 GHz oscillator, the radio and television transmissions, and the sound waves, the wave speed v can be found from $v = f\lambda$. Later experiments will measure the speed of some radiations directly. If the wave speeds of some radiations turn out to be the same, that would count as evidence in favour of their belonging to a common family. So the task in the experiments in 4.1 is to try to observe superposition effects, to devise one experiment in which it will be possible to measure the wavelength with reasonable accuracy, and, if practicable, to calculate the wave speed.

Experiment

4.1a Superposition of 1 GHz radio waves

The apparatus is quite simple, and all of it except the oscillator is in the open. If the oscillator output were looked at with a good enough oscilloscope, it would be seen to consist of very rapid electrical oscillations (a thousand million each second). The amplitude rises and falls in size more slowly, perhaps a hundred times a second, and as a result an amplifier and loudspeaker connected to the receiving aerial (and its diode) give an audible sound.

The transmitting aerial is a pair of rods – a dipole – each joined to one lead from the oscillator. At one moment, the oscillator pushes charge onto one rod while like charge flows off the other. Then the charge flows reverse, so that the rods are alternately charged positively and negatively. In fact, they are like the plates of a capacitor that happens to have been opened out at one end. The receiving dipole is similar. There is a diode connected to the rods, so that if electrical oscillations appear in the rods, there can be a one-way flow of current through an ammeter connected across the dipole. And such currents can be detected.

The system is clearly electrical at the transmitting and receiving ends, and the radiation in between could be electrical in nature. (*Must* it be?) There will be an electric field near the transmitting aerial if the rods become charged as described, and this field will rise and fall rapidly. Theory, looked at later in the course, says that the radiation does indeed involve fluctuations of electric field travelling from one dipole to the other.

Students should find out what things transmit the radiation (Perspex, hardboard, and other insulators); and what things stop or reflect the radiation (metal screens, though radiation will appear on the other side of a *small* screen).

Interference (superposition) effects will show up because of radiation scattered by students' hands, arms, or bodies. Several arrangements for measuring the wavelength are possible, including moving a reflecting plate behind the receiving aerial along the line joining the two aerials.

An audio amplifier and loudspeaker make a sensitive detector, effective up to several metres, though the risk of picking up someone else's transmission is then high, and the number which can be set up in one laboratory is small. A self-contained transistor amplifier and loudspeaker unit is best, as the whole apparatus may then be taken anywhere without need for a mains outlet.

The oscilloscope is about as insensitive a detector as the microammeter, and is rather less convenient.

Post Office regulations will prohibit the use of the apparatus if it is a nuisance, particularly if it causes interference on television channels. It is desirable to use a television set to discover the range at which there is interference, which will depend on the local frequencies, and the nature of the surrounding buildings. A good safety margin should be allowed.

Possible experiments

Many experiments are possible. The most important are indicated in figure 2, and are measurements of wavelength from a path difference, which can follow directly after tests of absorbing and reflecting properties of metal sheets, books, hands, etc. Students should be asked to devise some method of measuring the wavelength; it does not matter what the method is.

Faster groups may be encouraged to try further experiments, and all may profitably be allowed some time to play with the equipment.

Some could look at one or two of these things:

Directional properties of the dipoles.

Beaming the radiation with a curved plate behind the transmitter.

Polarization – the effect of 'crossing' the dipoles.

Effect of a 'reflector' as in a television 'H' aerial – a rod a little over half a wavelength long placed a quarter wavelength behind the receiving dipole.

Effect of a 'director' – a rod a little less than half a wavelength long placed a quarter wavelength in front of the receiving dipole. (A director makes the system less sensitive at large angles than at small angles to the line from dipole to director.)

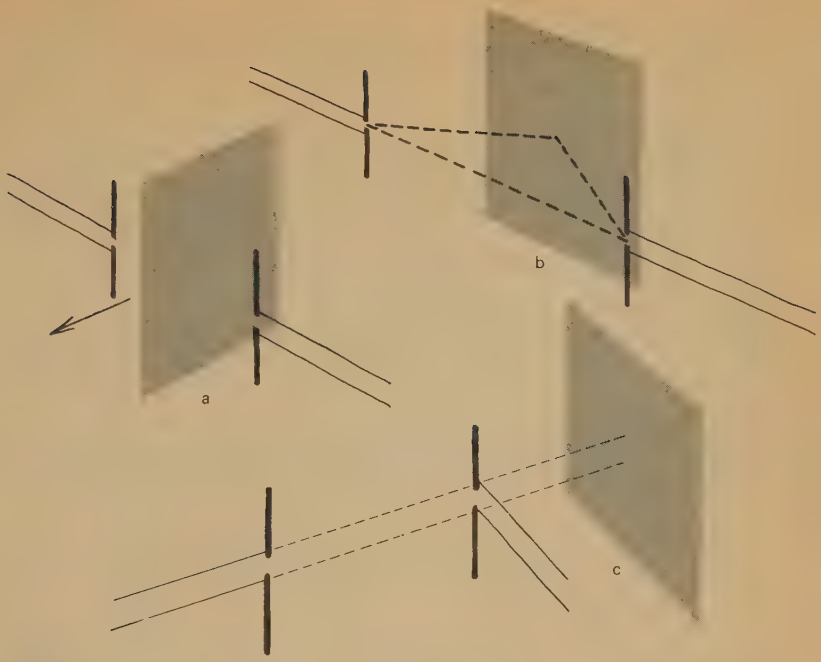


Figure 2
Simple experiments with 1 GHz radio waves.
a Absorption. **b** Reflection and path difference measurement of wavelength. **c** Another simple path difference experiment.

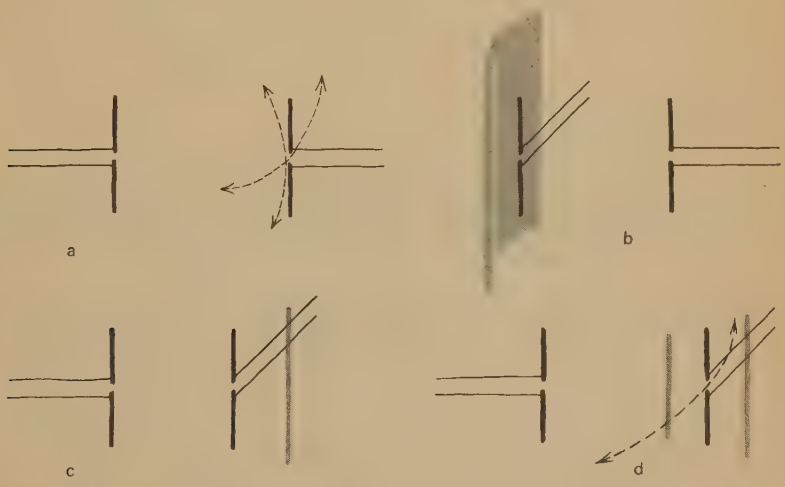


Figure 3
Further experiments with 1 GHz radio waves.
a Directional properties. **b** Reflection by curved mirror. **c** Effect of 'reflector'. **d** Effect of 'director'.

Experiment

4.1b Superposition of microwaves

184/1 3 cm wave transmitter

184/2 3 cm wave receiver

1053 metal reflector (about 0.3 m square) 2

1053 narrow metal plate 60 mm wide)

181 general purpose amplifier

183 loudspeaker (if not with above)

1002 microammeter (if not incorporated in receiver)

1045 diode probe for microwave experiments

1000 leads

A receiver made of a diode in a waveguide behind a horn is more sensitive and more directional than a simple diode on its own, but is more complicated. For simple student experiments, the single diode is best, and is essential for many experiments, such as that shown in figure 4 *a*.

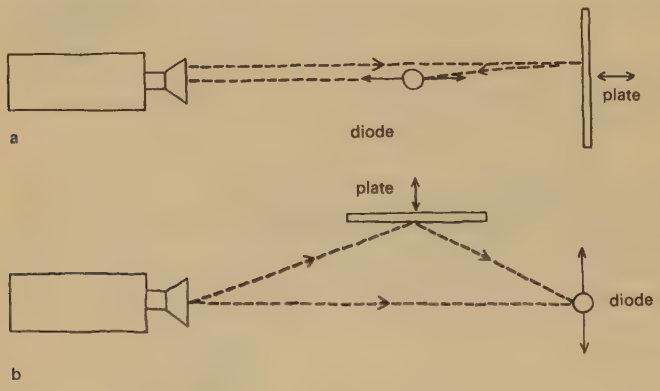


Figure 4
Simple superposition experiments with microwaves.

Students should be shown how to adjust the reflector voltage for maximum signal, and how to switch the transmitter power supply from the unmodulated position used for detection with a meter, to the modulated position used for detection with an audio amplifier. A portable transistor amplifier incorporating its own loudspeaker is best.

In case it arises, teachers should know that because of the phase change on reflection, if the diode in figure 4 *b* is moved very close to the reflecting plate the output is a minimum even though the path difference is zero.

Velocity of microwaves

Experiment 4.4, in which the speed of microwaves is measured directly, is a difficult demonstration which might be attempted.

Experiment

4.1b Superposition of microwaves

Here students can do a wide variety of experiments, if they are provided with screens that can reflect or block the radiation, or can be formed into single or double slits. It is probably best for them to work with the simplest detector – a diode and a meter. For weak signals, the diode will have to be connected to an audio amplifier and the signal modulated by the internal device that effectively switches it on and off 100 or 1000 times a second. Interference effects will inevitably show up, and again the wavelength can be measured in several ways.

It is worth while leading students to set up a two-slit experiment with which they measure the wavelength by direct measurement of the distances from each slit to the diode.

Figure 5
A microwave two-slit experiment.

In the experiments with microwaves, the frequency cannot reasonably be supposed to have been measured, so the velocity of microwaves cannot be calculated from the wavelength.

Optional experiment

4.1c Superposition effects with v.h.f. radio transmissions or u.h.f. television transmissions

portable radio set which receives v.h.f. transmissions

television set which receives u.h.f. transmissions

1077 television aerial

1062 drum of coaxial cable

1053 metal reflector

501 metre rule

The basic experiment is simple; to arrange a flat reflector facing the transmitter and locate positions of maximum and minimum reception at varying distances in front of the reflector. The difficulties of doing so are different for the two kinds of transmission.

V.h.f. radio

The direction of the station needs to be found. The table of stations and frequencies given in Appendix A can be used, together with a map. If the transmission is horizontally polarized (dipole aerials horizontal), the station lies along the direction of the dipole rods when they are rotated until the signal is a minimum.

If the radio set has a rod aerial, this can be used, or a dipole can be improvised from wires taped to a wooden bar. The wires should each be about a quarter wavelength long, and be connected by coaxial cable to the set.

The best radio sets have feedback circuits designed to keep the output volume constant over a wide range of input signal strengths, so making the experiment difficult. A cheap little radio is best for our purposes. A good one can be used more successfully if it is deliberately tuned slightly 'off' the station, and is best if the batteries are almost used up.

U.h.f. television

Details of the frequencies allotted to the various channels are given in Appendix A. The direction of the station can usually be found by looking at aerials on nearby houses, or by experimenting with a portable aerial.

If any signal at all can be obtained with an improvised dipole aerial, this is likely to be better for superposition experiments than a more complicated 'Yagi' aerial with a reflector and many director rods, partly because the dipole will be better at detecting radiation reflected to its back as well as that coming directly to it, and partly because the set will be receiving a smaller signal, and feedback circuits designed to compensate for signal variations will be less effective. The best results are likely to be got if the best picture obtained is of very poor quality.

An extension for fast students involves the ghost images which appear when a signal arrives, after reflection, some time after the main signal. For a 625 line set, at 24 pictures a second, the beam scans one line of the picture in 1/15000 s. A ghost image about one-tenth of the picture width out of place is delayed by 1/150000 s, corresponding to a path difference of about 2000 m. If the object responsible for the ghost can be identified, the speed of the waves can be estimated, using a map to determine the path difference.

Optional experiment

4.1c Superposition effects with v.h.f. radio transmissions or u.h.f. television transmissions

There is much to be said for letting one or two students devise ways of showing that the very familiar 'waves' that carry broadcast signals do have the essential property of waves: superposition effects. (Students who speak of 'interference' in this context should be clear that the word does not mean the leaking of signals from one channel into another or the noise from domestic electrical equipment that sometimes spoils reception.)

V.h.f. radio

The v.h.f. band with frequency about 90 MHz, wavelength 3 m if the wave speed is 3×10^8 m s^{-1}, is suited to work out of doors with a portable transistor radio having its own rod aerial. If a large reflecting screen can be arranged to face the transmitting station, the radio set can be carried about in front of it and positions of minimum signal located. If the frequency is known from the published value for the station or from the setting on the receiver tuner, and if the wavelength is known from the path difference measurements, the speed can be calculated.

U.h.f. television

The television set will normally have to be indoors, and it is possible to work indoors in a large enough room with a portable aerial or with an improvised dipole aerial. The direction of the station can be found by looking at aerials on house tops, which point towards it. The wavelength is only some 0.3 m, so reflecting sheets of modest size will serve. The effects of the walls of the building are likely to complicate matters, and ambitious students might run an aerial out of doors on a long length of coaxial cable.

Figure 6
Transmitter direction finding with a 'Yagi' aerial.

Reflecting screens

For v.h.f. radio a screen several metres each way is needed. Such a screen can be improvised from chicken wire or strips of aluminium cooking foil hung on a wall facing in the right direction. But the easiest solution is to look for a wire mesh fence which happens to face the right way. Sometimes a brick wall will serve, especially if it is wet after rain.

For u.h.f. television signals, the screen need only be about a metre each way, and is easily improvised from metal sheet, wire mesh, or metal foil.

Experiment
4.1d Superposition of light waves

Either two-slit interference: see Nuffield O-level Physics, *Guide to experiments V*, Experiment 105; *or* interference by reflection.

Use, for example:

52	Worcester circuit board kit
97A	microscope slide
1053	Plasticine
24	hand lens
192/1/2	red and green filters

Figure 7 shows one simple way of producing interference by reflection in normal room lighting. A flashlamp bulb stands on a circuit board so that its filament is horizontal, and at right angles to a line joining it to a microscope slide. The slide is horizontal, held at the height of the filament above the bench on a lump of Plasticine. Keep the slide's top surface clean. The slide should be 0.5 to 1 m from the lamp.

To find fringes, look along the slide at the lamp, and tilt the slide a little until the image of the filament is as close as possible to the filament seen directly. Then look with the hand lens at the back edge of the slide, getting this edge in focus. Equally spaced bright and dark fringes should be seen. Tilting the slide a little either way will increase or decrease the fringe spacing, and may improve the quality of the fringes first seen. Red and green filters can be used to see more fringes, and to note the change in their spacing with colour. Hold the filter between the slide and the lens. If the hand lens is taken further back from the slide, the quite different fringes due to diffraction at a straight edge may be seen. These are less sharp and are not equally spaced, being broadest near the edge producing them.

There are many other ways of arranging the experiment, and teachers will probably need to provide some written instructions to suit the method they favour.

The apparatus can be in a vertical plane, allowing more freedom of movement but more scope for misalignment. The lamp may be a 12 V 24 W lamp two or three metres from the reflecting surface. The reflecting surface can be a flat glass block or a strip of mirror. (If a mirror is used, make it clear that the reflection is off the top, not off the silvered surface.)

Once they have set it up, a pair of students can easily show the experiment to the rest of the class.

Experiment

4.1d Superposition of light waves

As students should know already, light, too, can be made to show interference (superposition) effects. Some empirical reminder may be useful, especially if it brings out the fact that the wavelength is much smaller than in the other experiments in this group.

For light, unlike the radiation from the 1 GHz oscillator, there is no way of knowing that the source is oscillating. The only reason for thinking that light is an oscillating something-or-other is the fact that beams of light have the characteristic property of a wave motion.

Students who missed a two-slit interference experiment at O-level should certainly set one up now. Others may like to try interference between light reflected from a flat surface at a shallow angle and light from the same source coming directly to the eye. Measurement of wavelength is not convenient in this last experiment; in any case, there are better ways of measuring the wavelength of light. Even if the wavelength were measured, the speed could not have been calculated as the frequency is not known. But the reflection experiment has the virtue that it is close in kind to the experiments done with radio and microwaves.

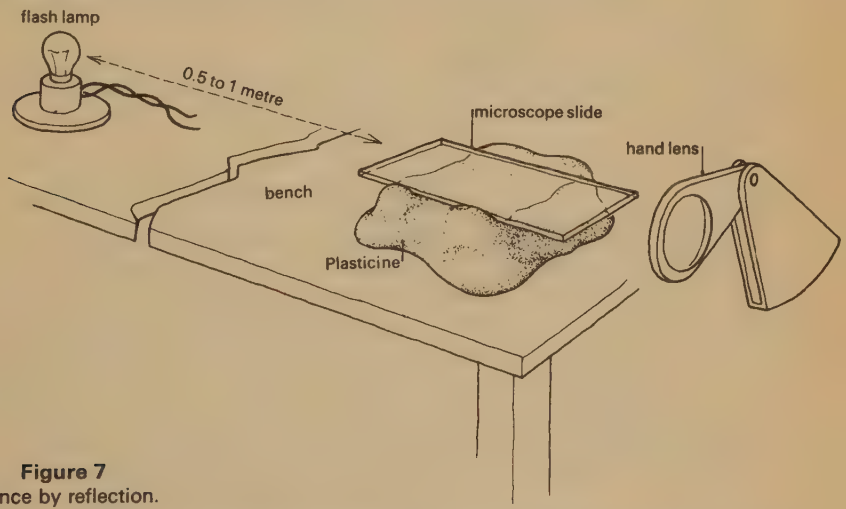


Figure 7
Interference by reflection.

It may interest some students to notice that the edge of the reflecting surface lies where a dark fringe would be. They may have noticed a similar effect in experiment 4.1b, with microwaves reflected from a metal sheet. As in that case, there is a phase change on reflection, so that the waves superpose destructively even though the path difference is zero. The point is not one to be striven for.

Experiment

4.1e Superposition of sound waves

1009 signal generator
157 microphone
158 class oscilloscope
1035 pre-amplifier
183 loudspeaker *2*
1053 metal reflector
501 metre rule
1000 leads

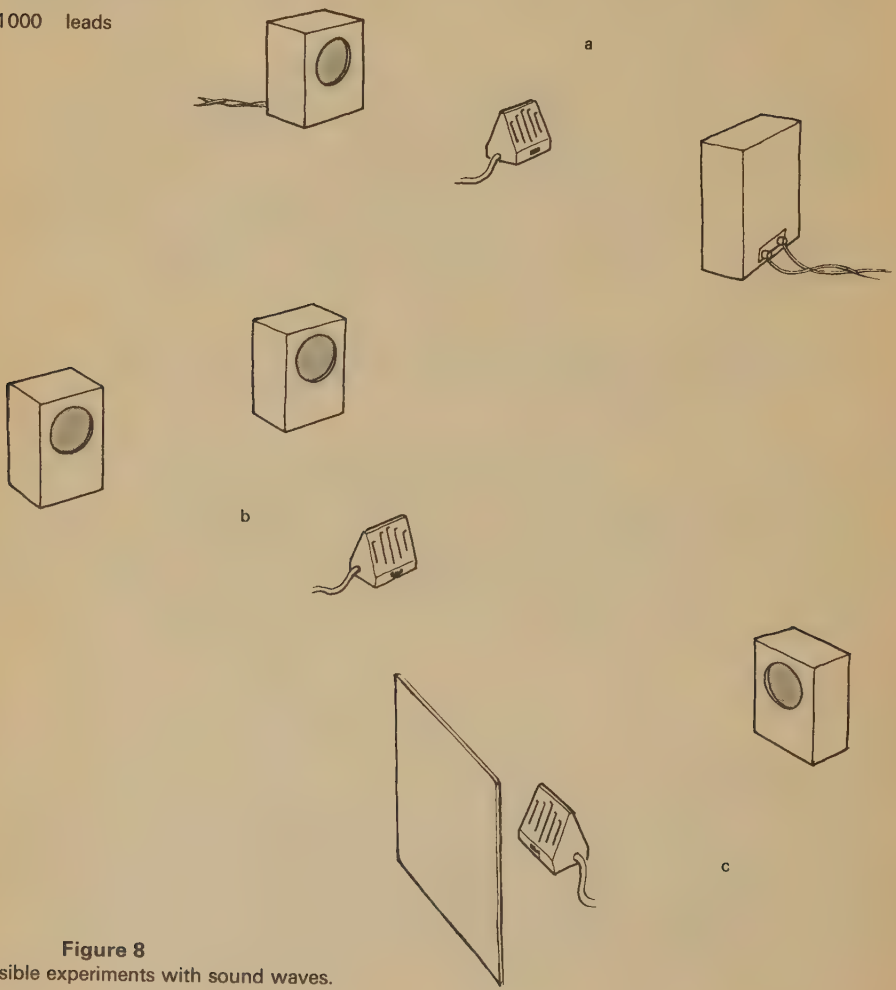


Figure 8
Possible experiments with sound waves.

Students should be reminded of the particular problems of optical superposition experiments. The light must come from one source, as the atoms in two filament lamps cannot be driven in any fixed phase relationship. The source must be narrow and far off, or the bright and dark bands due to light coming from one part of the filament will overlap those of light coming from other parts, producing a uniform illumination. The wavelength is small, so the sources must be close together to produce a pattern big enough to see.

Experiment

4.1e Superposition of sound waves

This experiment is mainly valuable for the contrast with electromagnetic waves. The frequency for a comparable wavelength is much lower, because sound travels about a million times slower than electromagnetic waves.

As with the other experiments (except 4.1d), students can usefully be asked to devise their own arrangement, being provided with a pair of loudspeakers and one or more reflecting surfaces.

It is helpful to reverse the argument about speed, frequency, and wavelength, and ask them to choose a suitable frequency to give a wavelength of a convenient size, say 0.1 m, knowing the speed of sound.

If two loudspeakers are used, a good point can be made out of the effect of reversing the connections to one speaker, so reversing the phase of one and turning maxima into minima and vice versa.

Summary: superposition of waves

All students should have handled the 1 GHz equipment, and should see demonstrations or hear reports about the other experiments that have been tried. The points to be made in discussing the whole group of experiments are as follows.
Wave motions show superposition behaviour, with regions where waves are in step, giving high intensity, and regions where they are out of step, giving low intensity.
Radio waves, microwaves, and light are thought to be waves because they show superposition effects, not because anyone can see anything oscillating.
If radio waves, microwaves, and light do belong to one family, they might travel at the same speed. There should now be evidence that radio waves travel at the speed 3×10^8 m s^{-1}, reported in books as the speed of light.

Further experiments will attempt measurements of the speed of light and perhaps the speed of microwaves.

Many variations are possible, as shown in figure 8. The loudspeakers may face each other some 0.5 m apart, and the microphone be moved about between them (variation *a*), or they can be placed side by side two or three wavelengths apart so that the region in front of them contains an interference pattern (*b*).

A single loudspeaker can be used with a reflecting plate in front of it, the microphone being moved between speaker and reflector while facing the reflector (*c*).

Instructions on handling the equipment (as distinct from designing the experiment) may be needed and can save time. Useful points are:

How to handle the controls of the oscillator.

To connect speakers to the low impedance output of the oscillator.

To have speakers in parallel, generally.

To keep the intensity low, and use a pre-amplifier between microphone and oscilloscope, so that room reflections are less troublesome and the noise of the experiment is less of a nuisance to others.

The *Students' laboratory book* contains notes on the use of the oscilloscope. It is worth reminding students that the oscilloscope is to be considered a vital tool of the physicist's trade, and that they ought to learn to use it with fluency and skill.

Optional extra demonstration using the double beam oscilloscope (item 1007)

Two microphones, connected to the two inputs of the oscilloscope, both listen to the same loudspeaker. The microphone connected to the channel not triggering the time base is moved along the line between itself and the loudspeaker. The two traces then change phase relatively to one another. The experiment is revealing, in showing what phase means, what wavelength means, and what a double beam oscilloscope does.

Film

'The velocity of gamma rays'

This 16 mm sound film was made in conjunction with the Advanced Physics Project. It shows the timing of short bursts of gamma rays across a distance of a few metres. The gamma rays were produced by letting short bursts of particles from a cyclotron fall on a metal target. The speed of gamma rays is shown to be 3×10^8 m s^{-1}.

The film is not essential to the course, but is useful here or later on in Unit 5, *Atomic structure*, when the nature of gamma rays is again raised. It is of interest, too, because it shows what it is like inside a working nuclear physics research laboratory.

For details of this film and other films and loops recommended in this *Guide*, see the list on page 138.

A family of waves travelling at the same speed

What does a book mean by the 'electromagnetic spectrum'? The idea is that radiation from long wavelength radio at one extreme to gamma radiation at the other, through short wave radio, microwave, infra-red, visible, ultra-violet, and X-ray radiation, are somehow all 'the same', differing only in wavelength. The word 'spectrum' is used by analogy with the visible spectrum produced by a prism or diffraction grating, in which the colours are spread out according to wavelength. Figure 9 suggests the fanciful idea behind this use of words.

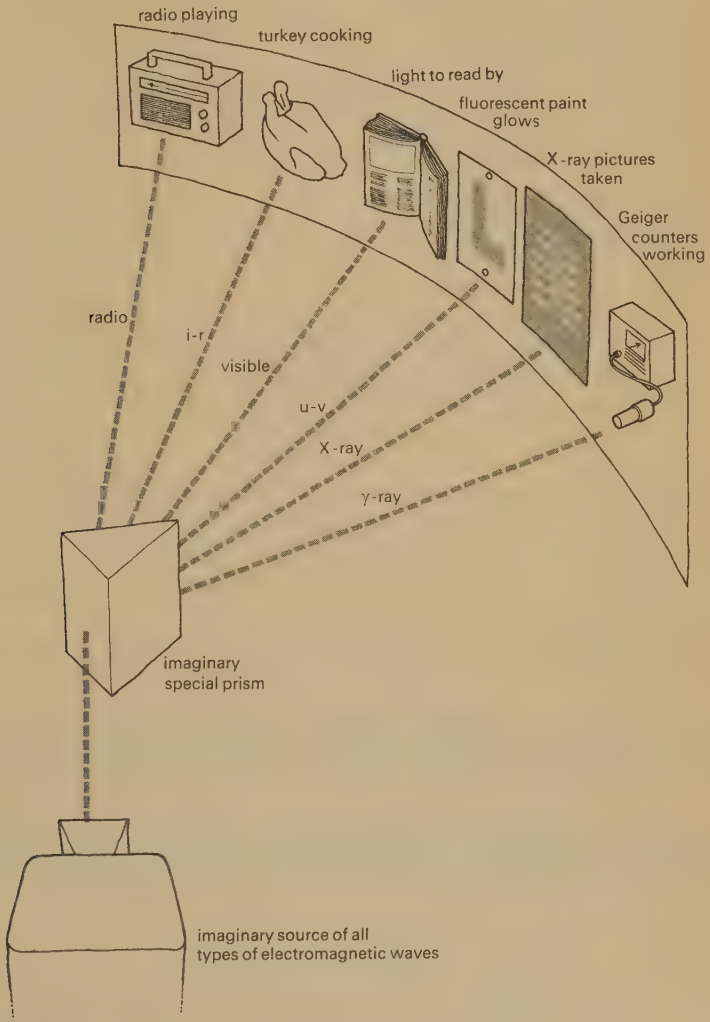


Figure 9
Fanciful illustration of the meaning of the electromagnetic 'spectrum'.

Demonstration

4.2 Infra-red and ultra-violet radiation

1068 parallel beam projector

59 l.t. variable voltage supply

69 high dispersion prism

68 phototransistor

1033 cell holder with one U2 cell ($1\frac{1}{2}$ V)

1003/1 milliammeter (1 mA)

1046 infra-red and ultra-violet filters

1054 printing paper (P153) developer, fixer
or
1053 fluorescent paper (green)

1053 screen of non-fluorescent white paper and support

1000 leads

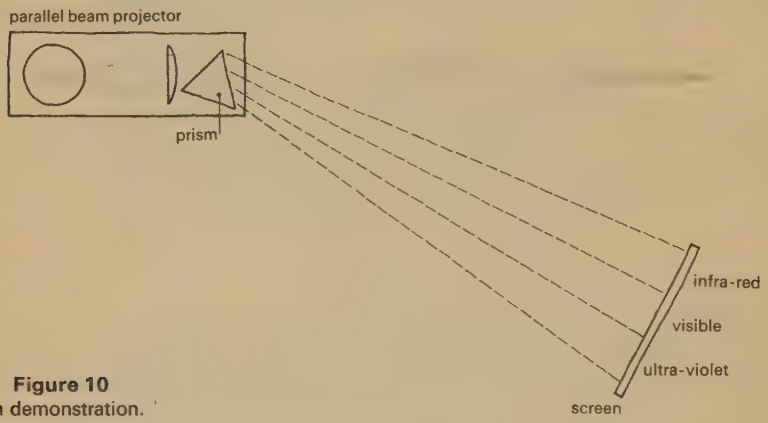


Figure 10
Spectrum demonstration.

The parallel beam projector and prism are used as shown in figure 10 to cast a spectrum onto a screen made of a piece of board about 0.3×0.2 m to which is pinned a sheet of non-fluorescent white paper set up about 0.5 m from the projector. As usual, adjust the lamp until its image would be in focus on the screen, and rotate the prism to the position of minimum deviation.

The white paper, which could be blotting paper, can be tested for lack of fluoresence with an ultra-violet lamp (item 189) and the ultra-violet filter, in a dark room. For demonstration purposes, the lamp may be overrun by up to 30 per cent. It should have a linear filament.

But what evidence could one offer to support the view that these seemingly different radiations are all 'the same'? Students have just seen that both light and several sorts of radio wave have wave properties, and Unit 1 gave the suggestion that X-rays have them too. Suppose all these sorts of radiation had wave properties – would that be enough? (No, they could be different sorts of wave. Sound is distinctly different from light.)

Another line to follow would be the speed of the waves, and it does turn out that all these waves travel at the same speed, the 'speed of light'. Not all can be measured in the school laboratory, but one or two can, and these will have to stand proxy for measurements at other wavelengths.

In addition, there is a film showing a measurement of the speed of gamma rays by a direct timing method, which is useful as representing the very short wave end of the spectrum.

The idea of the electromagnetic spectrum can be reinforced by next taking a brief look at a spectrum containing visible, infra-red, and ultra-violet radiation.

Demonstration

4.2 Infra-red and ultra-violet radiation

Another line of argument that lends some support to the 'family' view of electromagnetic radiation is the degree of overlap in properties shown by different parts of the spectrum.

For example, long wave heat radiation from a hot object can warm things up, and so can the radiation with wavelength of the order of centimetres or millimetres used in modern radar, and indeed also in microwave ovens. But these radio waves in turn share many properties with other radio waves of much longer wavelength.

The overlap between visible and infra-red, and between visible and ultra-violet, can be shown in the school laboratory.

A phototransistor will detect radiation in a spectrum from a lamp across the visible and well beyond the red, in the infra-red region where the most noticeable property is the warming up of an object held in the radiation.

Photographic paper is affected over much of the visible spectrum, but also well beyond the blue, in the ultra-violet region where another noticeable effect is the fluorescing of certain paints.

Ultra-violet radiation of wavelength rather shorter than that ever produced by a lamp and transmitted by a prism will ionize air and eject electrons from metals, but so will X-rays and gamma rays; indeed the latter are detected in a Geiger tube by means of the electrons they produce inside the tube.

Infra-red

Students may already have seen this at O-level (Nuffield O-level Physics, *Guide to experiments II*, experiment 97).

Connect the phototransistor to the dry cell and milliammeter in series, and put it just in front of the screen. If the projector is rotated to sweep the spectrum across the phototransistor, a peak response will be found beyond the visible red region.

With the transistor in the region beyond the peak, the effect of the filters can be shown.

Ultra-violet

Do **a** or **b**. **b** is quicker, but shows less of the ultra-violet.

a Pin a strip of P153 daylight printing paper to the screen and expose it to the spectrum for several seconds, marking the limit of the visible blue-violet with a soft pencil. Subdued incandescent room lights may be left on. Develop the paper in front of the class, when it will be seen that the paper is blackened well beyond the visible region. The dyes in the paper make it insensitive to parts of the visible spectrum.

b Pin the strip of fluorescent paper so that the lower half of the spectrum falls on it, the upper half still falling on the white paper. In a darkened room, some fluorescence can be seen beyond the visible if the lamp is overrun. Much of the fluorescence is in the visible blue-violet, but the difference is shown up by use of the ultra-violet filter to remove much of the visible region.

Long experiment

4.3 Measurement of the speed of light

1032	speed of light apparatus
503	retort stand base (large) *3*
504-5	retort stand rod, and boss *2*
21	compact light source
27	transformer
176	12 volt battery
1055	reversing switch
38	single pulley
507	stopwatch or stopclock
501	metre rule
1000	leads

The *Students' laboratory book* contains instructions for doing this experiment, with apparatus developed by the Nuffield Advanced Physics Project. This apparatus uses a curved rotating mirror and a curved distant mirror. Other forms of the apparatus are available, and may use slightly different optical systems incorporating plane mirrors and lenses. The instructions will need modification for such apparatus.

It is suggested that the measurement be treated, along with measurements of G, e, etc., as a long experiment to be tackled by one group of students at some convenient time. Indeed, it may already have been done, along with earlier long experiments.

This is one of the least arduous of the long experiments, and need occupy no more than a double period. Students who cannot do it in this time need help, very probably with the adjustment of the optical system. Question 8, in the *Students' book*, helps with the calculation.

The evidence is comparable with that to be seen in some family photographs, showing everyone from grandparents to the latest grandchild. Two of them may share the shape of a nose, two or three a common build, and others may have the same colour of hair. No two are exactly alike, but each shows his or her relationship with another. The evidence is not compelling, but is a helpful support to other arguments.

Long experiment

4.3 Measurement of the speed of light

One or two groups of students can be sent off to measure this fundamental constant, using a fast spinning mirror method. Once the apparatus has been set up, all the class can be shown the evidence for a finite speed of travel given by the shift in the image made by a beam of light reflected twice at a rotating mirror with a long path between the reflections. This might be done as a demonstration before measurements are made.

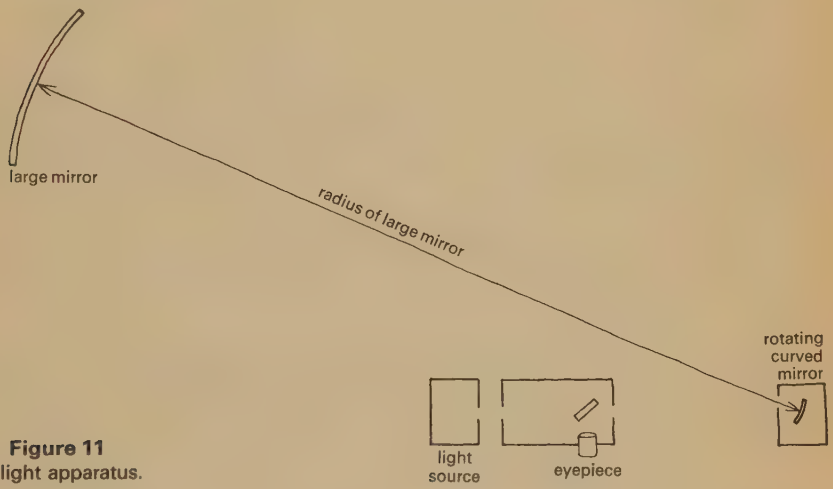


Figure 11
Speed of light apparatus.

Optional demonstration (or long experiment)

4.4 Measurement of the speed of microwaves

184/1	3 cm wave transmitter
184/2	3 cm wave receiver
181	general purpose amplifier
	and
183	loudspeaker (if not with the above)
1031	200 kHz pulse generator
1014	wax lens *2*
1035	pre-amplifier
1007	double beam oscilloscope
1033	cell holder (with four U2 cells) *2*
1053	metal screen *2*
1065	big mirror
501	metre rule
1000	leads

The lengthy instructions for this demonstration appear in Appendix B. Some schools will not have the necessary large empty space in which to attempt it. Even given the space, the experiment is not easy, and requires much skill and patience to arrange successfully. It could be regarded as a project to be given to an interested group of students, or as a set piece to be shown on a special occasion when time and space might be made available. Details are also given in the *Students' laboratory book*.

The alternative experiment from Unit 8 is 8.10, *Speed of a pulse in a coaxial cable*. It presents no practical difficulties, and should be seriously considered for showing now.

Students' book

Question 8 is about measuring the speed of light and should enable students to argue out the means of calculating the speed from experimental measurements; 9 to 14 are more general questions about wave speed, particularly about uses, such as radar, of knowledge of the speed of waves.

Discussion of evidence

The purely experimental evidence is suggestive, not compelling. 'Everyone already knows' about the electromagnetic spectrum, but a student ought not to think that the discussion is an attempt to give convincing reasons why the common view is correct. Rather, its purpose is to show that the common view must stand or fall on the evidence, not on the frequency of its repetition in books. The incompleteness and insufficiency of the evidence are points to be brought out, not concealed. Too many students are surprised to find that physicists actually search for flaws in arguments or weaknesses in evidence, and are glad to find them.

It may also be possible to bring out that this has been solely an empirical study, not backed by any theory, despite the difficulty of saying to students at this stage what a theory would be like. But it should be clear that the facts looked at do not include or imply descriptions of travelling electric or magnetic fields or any other pictures students may have seen in books.

There is an advantage in students knowing more than the facts support, for the difference between that which they know and that which they can support with evidence, can be brought out.

Optional demonstration (or long experiment)

4.4 Measurement of the speed of microwaves

The experiment consists of timing the delay suffered by a pulse of microwaves sent down a long room to a mirror and back again. The time is found by displaying the outgoing and returning pulses on an oscilloscope, using the oscilloscope time base as a clock. Even if the experiment is not tried, it is worth mentioning the principle, and pointing out that radar sets use the method daily. Indeed, there are now radar sets in use for land surveying, and in principle one of these could be used over a measured base line to provide a value of the speed.

Alternatively, the measurement of the delay of a pulse travelling along a coaxial cable is worth bringing forward from Unit 8. Besides illustrating the principle and giving a speed differing from that of light only because of the nature of the insulator within the cable, it raises the interesting question of why the electric light comes on (almost) as soon as the switch is closed, even though the electrons in the wires move very slowly (as seen in Unit 2). (The voltage or field under whose influence the electrons begin to move sweeps very quickly round the circuit, setting up wherever it has gone a current carried by slowly drifting electrons.)

Summary: electromagnetic waves

It is good here to look back and to look ahead. It has been shown that some members of the electromagnetic wave family are waves. Evidence also exists for other parts of the spectrum which were not sampled for tests of wave properties. The wave properties are less obvious at the gamma ray end of the spectrum; indeed gamma rays seem remarkably like particles if one listens to the clicks of a Geiger counter detecting them, though this may seem to say more about the Geiger tube than about the gamma rays. This puzzle, noted already for X-rays in Unit 1, will be of great importance in Unit 10, *Waves, particles, and atoms.*

That the waves are electromagnetic has not been shown, though some are made and detected by obviously electrical devices. Unit 8, *Electromagnetic waves,* will look at theoretical reasons for this name, and will show how the speed of all members of the family has the same value, which can be calculated theoretically.

Meanwhile, there is evidence that the speed is the same. Table 1 collects together some measured values of the speed at wavelengths differing by a factor of nearly 10^{15}. Note the constancy of the speed (and the varying accuracy).

Photons

The relationship $E = hf$ will be introduced in Unit 5, and used again in Units 9 and 10.

Table of speeds

See table 1.
Appendix C gives further evidence, from astronomy, of the constancy of the speed.

Textbooks

PSSC, *College physics.* Chapters 3, 7–9.
PSSC, *Physics,* 2nd edition. Chapters 11, 16–18.
Both these discuss light waves, interference, and simple diffraction.

Arons, *Development of concepts of physics,* Chapter 22, is suited to revision of ideas of superposition and interference.
Holton and Roller, *Foundations of modern physical science,* Chapter 29, is a brief, general account of the history of experiment and theory relating to electromagnetic waves.
Rogers, *Physics for the inquiring mind,* Chapter 10, is brief but good on superposition and on wave motion.

Further reading

Sanders, *The velocity of light,* reprints papers by Michelson and others.

One of the following could interest a student wanting to find out more about radio astronomy. The first is a paperback, the others are reprints.
Graham Smith, *Radio astronomy.*
Heeschen, 'Radio galaxies'.
Westerhout, 'The radio galaxy'.

Slides

Slides 4.5 (1 to 9), are a series describing the Decca Navigator system, which depends upon phase differences between signals from transmitters. See the list of slides (page 134) for details and a suggested commentary.

Students' book

Question 18 can be used as the basis for discussion of the uses of electromagnetic waves.

The speed of electromagnetic waves

Wavelength/m	Speed/10^8 m s^{-1}
6.4	$2.997\,8 \pm 0.000\,3$
1.8	$2.997\,95 \pm 0.000\,03$
1.0	$2.997\,92 \pm 0.000\,02$
1.0×10^{-1}	$2.997\,92 \pm 0.000\,09$
1.2×10^{-2}	$2.997\,928 \pm 0.000\,003$
4.2×10^{-3}	$2.997\,925 \pm 0.000\,001$
5.6×10^{-7}	$2.997\,931 \pm 0.000\,003$
2.5×10^{-12}	2.983 ± 0.015
7.3×10^{-15}	2.97 ± 0.03

Table 1
Adapted from French, A. P. (1968) Special relativity, *Nelson.*

This table appears in the *Students' book*, page 113.

Part Two

Mechanical waves

Time: about a week

Difficulties of teaching about mechanical waves

Inevitably, this part will have to be confined to the simpler sorts of wave, and to only some of them. Students will be asked to do rather simple experiments with rather simple apparatus, and may grow restive at the smallness of the enterprises proposed. So some words about the usefulness of knowledge about such waves, and about the place they have in the course, may help.

So far as the course is concerned, the vital point is that a physicist hopes to be able to explain the behaviour of a particular kind of wave in terms of more basic ideas. Unless students see this, the work of Unit 8 on explaining electromagnetic waves may seem both vague and arbitrary.

Note to teachers who start at Part Two

Students with a smaller background of experience of visible waves may do better to start at Part Two, combining it with work on ripple tanks selected from the third year of Nuffield O-level Physics. For them, the experiments of the group 4.5 will serve to show how waves superpose, and to illustrate the ideas of frequency, wavelength, and wave speed.

Students' book

Questions 19 and 20 provide revision of the dynamics needed in this part. Questions 21 to 23 introduce some simple aspects of wave motion, needed later on.

Group of experiments

4.5 Mechanical waves

It is suggested that most students be given 4.5a, making transverse waves on a long narrow spring. This is particularly good for bringing out what decides the shape of a pulse, showing that pulses superpose, observing reflected pulses and standing waves, and seeing that the tension is important in deciding the wave speed, but that pulse shape is not.

For variety, one or two groups could try the other wave devices.

4.5b, 'Longitudinal and transverse waves on a Slinky spring', is not quite so good as 4.5a for demonstrating transverse pulses, but allows longitudinal pulses of compression or expansion to be made.

4.5c, 'Transverse waves on a wave model made of trolleys and springs', allows the tension and mass to be varied separately, and is very good for drawing attention to the dynamics of the motion of particles in a wave. It is less good for pulse shape or superposition observations.

4.5d, 'Waves on water', is excellent for superposition observations. Water waves in a shallow trough can form the subject of many possible individual investigations, and interest might be sparked off by a quick look now.

Use of terms – wavelength, amplitude, frequency

Students should come to know these terms well enough to use them in describing what they see, and in understanding books, articles, or examination questions. The discussion following the observations made in experiments in 4.5 is an opportunity to re-introduce these terms. It would be wrong to set such strong store by correct use of terms that students are reluctant to join in discussion lest they use them wrongly. But their meanings ought to be explained.

Waves we can see

Part One dealt with invisible waves; Part Two is concerned with waves that can be seen or felt, like waves on water; waves on taut wires or on strings; sound waves; shocks or vibrations in buildings or in the Earth, and waves of compression and expansion passed along from atom to atom of a solid.

Such waves have practical importance for ship builders and harbour makers; designers of musical instruments or of telephone cables, electricity cables, and suspension bridges; acoustic engineers; architects, installers of vibrating machinery, aircraft engine designers, geophysicists, and many others.

There is no time to look at all these waves, but luckily there is no great need to, for arguments about why the wave travels, how fast it goes, and how and why something, whatever it is, moves up and down or to and fro, for just one or two waves, turn out to apply with little change to the other waves. The point is to show that by thinking hard about what is going on in a wave, it is possible to predict its speed, using only simple ideas about mass, force, and motion. Later, Unit 8 will show how the same job can be done for electromagnetic waves, this time using only ideas about electric and magnetic fields.

Group of experiments

4.5 Mechanical waves

Before one can argue about what is going on in a wave, one must look to see what is happening. Therefore, students are asked to observe as much as they can about some particular waves, mainly transverse wave pulses on stretched springs.

To begin to argue about the speed of waves, it will be necessary to have some idea of the answers to a number of questions:

Does the speed depend on the shape of the pulse – its height or length?

Does the speed depend on the spring – how could the speed be made larger or smaller?

Does friction make any difference to speed or pulse shape?

What decides the shape of a pulse?

What happens when pulses cross each other?

What happens when a pulse is reflected?

Can one pulse catch up another and pass it?

Experiment

4.5a Transverse waves on a long narrow spring

507 stopwatch or stopclock

501 metre rule

1013 long spring

Six long narrow springs are suggested for a class of sixteen, so that more than half the class can be engaged on this experiment. The spring is of the kind used by PSSC, and shown in photographs in PSSC *Physics,* Chapter 15, or PSSC *College physics,* Chapter 6. Because the spring is narrow and closely wound, the shape of pulses is easy to see. Nor do these springs become entangled as easily as do Slinky springs.

Each pair of students needs a long narrow space to work in, and it is usually best to work on the floor rather than on a bench. Corridors can be exploited to advantage.

Figure 12 illustrates a useful demonstration of superposition. Figure 13 suggests a way of showing that a standing wave is made from waves travelling in opposite directions.

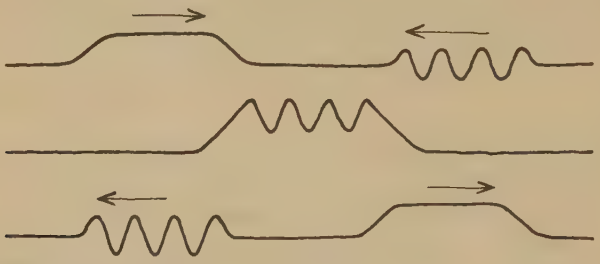


Figure 12
An experiment to illustrate superposition.

Slides

Slides 4.2 and 4.3, from the PSSC texts, show transverse pulses on ropes and springs, and illustrate superposition.

Experiment

4.5a Transverse waves on a long narrow spring

Encourage students to start with single hump-like pulses made by giving the end of the spring a single sideways flick. The speed does not depend on the amplitude of the pulse, and the pulse travels with essentially no change in shape. The speed increases as the spring is stretched. Students may be encouraged to suggest why, not because they will get it right, but because they may mention force, mass to be moved, or both (and both are changed by stretching the spring), and these will be important in later discussion.

Other important observations, to be encouraged by questions while experimenting goes on, and to be brought out in discussion, are:

Pulses superpose one on top of another, and pass 'through' one another without effect on either.

Pulse length depends on wave speed and on the rapidity of the motion starting the pulse.

Pulse speed does not depend on the rapidity of the motion starting the pulse (speed independent of frequency) nor on friction, but the latter decreases the pulse amplitude progressively.

Pulses are reflected at a fixed end, but the pulse is upside down (amplitude changes sign).

Continuous wave trains can be sent along and reflected. At particular frequencies the waves combine to form a spring oscillation pattern that does not travel (standing wave).

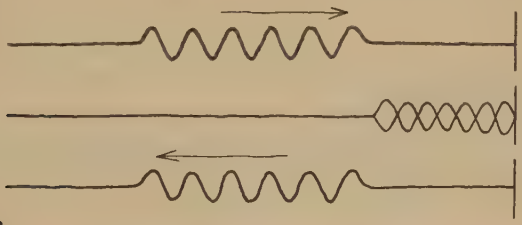


Figure 13
A standing wave produced for a short time as incoming and outgoing waves cross.

Experiment

4.5b Longitudinal and transverse waves on a Slinky spring

101 large Slinky spring

501 metre rule

507 stopwatch or stopclock

One or two groups could try the Slinky spring. A smooth surface on which to rest the spring is important for observation of longitudinal waves.

The following, perhaps as a demonstration, will help later theory. Pull one end of the spring sharply and keep on pulling, moving the end at a steady speed. Watch the stretched region of spring spread along to the far end, which feels no pull until the wave front reaches it. Note that in the stretched part, all the coils are moving slowly one way, while the wave front is travelling faster the other way.

The theoretical argument suggested for the wave speed considers a similar situation, and students will need to be able to distinguish between the wave speed and the speed of motion of the parts of the spring.

Experiment

4.5c Transverse waves on a wave model made of trolleys and springs

106/1 dynamics trolley *12*

2A expendable steel spring *44*

32 1 kg mass (or 12 more trolleys) *12*

507 stopwatch or stopclock

The idea for this model was taken from television programmes presented by D. C. F. Chaundy.

To connect the springs, arrange twelve dynamics trolleys close together, side by side and 'nose to tail'. Join each trolley to its neighbour by a spring at each end of the trolleys, and then pull at both ends of the model to stretch the springs, adjusting the middle trolleys so that all are equally spaced.

If the model is set up on a table with no edge it is easy for part of it to run off the side of the table, when the rest of the model inexorably follows! So it is best set up on a smooth floor, or on a surface provided with barriers along its edges.

Extra masses, whether laboratory masses or extra trolleys, are stacked one on each trolley in the model. Extra springs are placed with one extra one in parallel with each one in the model, keeping the length of the model the same.

Note for teachers on lumped wave systems

A system like this model, with the mass of the wave medium concentrated in discrete lumps with forces between each, does not behave in all respects like a smoothly spread out medium would do. The system is dispersive: the speed depends upon the wavelength when the wavelength is not much larger than the spacing between parts of the lumped medium. It exhibits 'cut off': waves of high frequency are not propagated at all. (Try moving an end trolley very rapidly to and fro. The next-door trolley oscillates a little, the next oscillates less, and there is something like an exponential decrease of amplitude along the system. No wave energy propagates down the system.)

These problems are discussed in The Berkeley Physics Course, Volume 3, *Waves*.

Experiment

4.5b Longitudinal and transverse waves on a Slinky spring

Observations of transverse pulses are similar to those in 4.5a. The slower speed of pulses on a Slinky is worth noting (due in part to the smaller tension). Longitudinal pulses should be investigated, especially to show that compression and expansion pulses go at the same speed. The motion of one part of the spring as the pulse goes past it should be looked at.

Standing waves of both sorts can be made.

Experiment

4.5c Transverse waves on a wave model made of trolleys and springs

The model is made of a row of trolleys linked by springs, as shown in figure 14, with the trolleys spaced out so that the springs are in tension.

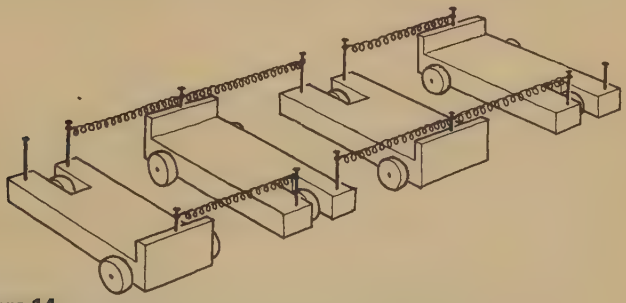


Figure 14
Transverse wave model made of trolleys and springs.

The mass of each trolley can be doubled by adding loads, and the tension can be doubled by adding extra springs. It is best to exploit this, and encourage students who try the model to concentrate on these dynamical aspects. If it is not used by students, the model can serve as a convenient demonstration around which to focus discussion of the experiments with waves on long springs.

Doubling the mass of each trolley reduces the wave speed; doubling the tension raises it. Both modifications change the speed by the same factor (actually $\sqrt{2}$) and both made together will restore the speed to its original value. Clearly, the wave speed depends upon how long it takes each part of the model to acquire some speed when forces act upon it, as the wave front arrives.

Optional extras for enthusiastic photographers

133 camera

171 photographic accessories kit

1054 film and monobath developer-fixer

134/1 motor-driven stroboscope

4A drinking straw

1054 printing paper, developer, fixer, slide projector

The trolley system is good for showing how the parts of a wave medium move. A stroboscopic photograph is taken along the line of trolleys, with two well-spaced trolleys carrying milk straw markers. A suitable pulse is made by accelerating an end trolley quickly, moving it sideways at steady speed for a time, then bringing it to rest sharply. The motion of the marked trolleys can be analysed from the photograph.

Experiment

4.5d Waves on water

1053 plastic guttering, 2 m long, 100 mm wide

1053 end stop for guttering *2*

533 bucket

100/2 wooden block from large rectangular tank

The guttering, both ends closed by end stops, rests on the bench. The sort of guttering that has a flat bottom needs no side supports to prevent it rolling over.

Formulae for the speeds of ripples and shallow waves are given in this *Guide*, page 55, and in the *Students' book*, page 112.

Refer interested students to Tricker, *Bores, breakers, waves and wakes.* This beautiful book could well stimulate many individual investigations. See also Barber, *Water waves*, which presents rather advanced mathematics very nicely.

Choice of a wave to analyse in detail

No compelling reason can be given for choosing to examine sound waves in solids in detail, showing how the wave speed can be calculated theoretically. Any wave will do, if students understand that the calculation is intended as a sample of all the possible calculations for other waves. Teachers may prefer the speed of a transverse wave along a thin flexible spring, string, or wire. But the wave we suggest has the advantage of linking up well with work on the force constants of atomic bonds in Unit 1 (and also in Unit 3, in work on sodium chloride). And perhaps sound waves may seem to students to be of more general interest than waves on ropes.

Optional experiment

4.5d Waves on water

Waves and ripples are sent along water in a long shallow trough. Because waves of many sorts can be made, and they travel slowly, the device is good for observing how wave pulses superpose, and pass 'through' one another without either being affected.

The rare student who has heard of group velocity, or one who wishes to try out formulae for the speeds of ripples or of shallow water waves, might get most from experiments with the water trough.

Theoretical calculation of the speed of a compression wave

There are many sorts of mechanical wave, of varying importance. The waves on the long spring are similar to waves on guitar strings, power cables, or aerial mast stays. Several books show how to calculate their speed. Then there are waves on stiffer rods or beams which can flex somewhat; important in xylophones, aeroplane wings, and suspension bridge roadways. The latter are rather harder to deal with theoretically.

Sound waves (that is, compression–expansion waves) in air, or transmitted through the structure of buildings containing vibrating machinery, or travelling through the earth from earthquakes are of interest and importance. (Both of these last two examples can also involve flexural and other waves.) Sound is chosen as one definite example to consider further.

It will be possible to give a theoretical calculation of the speed of sound in a solid, say steel; check it experimentally, and link the calculation with what is already known from Unit 1 about the stretching and compressing of steel. Similar arguments would give the speed of sound in air or in water. Because the vibrations in a sound wave are too tiny and too fast to be seen with the naked eye, another large-scale model is useful.

Demonstration

4.6 Longitudinal wave on a wave model made of trolleys and springs

106/1 dynamics trolley *11*

32 1 kg mass (or 11 extra trolleys) *11*

1080/1 compression spring *20*

1080/2 spring holder *20*

507 stopwatch or stopclock

501 metre rule

77 aluminium block

130/1 scaler

81 newton spring balance (10 N)

92X PVC covered insulated copper wire (26 s.w.g.)

52K crocodile clip *2*

1000 leads

The eleven trolleys are laid end to end in a row and each trolley is linked to the next by a compression spring, using a spring holder on each end of each trolley. (Some trolleys have a projecting front wheel. Such trolleys may have to be linked between their towing pegs with extended steel springs, item 2A. These springs snap shut when released, so the whole model must be held in tension, giving a less good representation, from the student's point of view, of atoms at an equilibrium distance which may be reduced or extended by a compression or expansion.)

The distance x (see figure 15) is measured with the trolleys in equilibrium. The spring constant k can be measured by pulling on one end of the row of trolleys with a spring balance, the other end of the row being fixed. There are ten springs in a row of eleven trolleys, so each spring is compressed by one-tenth of the distance the pulled trolley moves. The force recorded by the spring balance is the force in each spring, so the spring constant k is the spring balance reading divided by the compression of one spring.

Demonstrating waves

Show compression and expansion pulses. Show also, for use in the theoretical argument, a wave front made by pushing on an end trolley and then keeping it moving at steady speed, less than the speed of the wave travelling out ahead of it. It is important that students can distinguish between the speed of the trolleys in such a wave, and the speed of the wave front. Figure 16 shows the idea. It may help to show such a wave with a Slinky as well, when the wave speed can be slower.

Measuring the speed of the wave

See page 48. The theoretical argument will be helped by showing qualitatively or roughly quantitatively the effect of doubling the mass of each trolley and of adding a second spring in parallel with each one already between pairs of trolleys.

Students' book

Question 24 is a key question in this Part. It goes step by step through the argument for the wave speed of compression waves. Students can try it first, and then go through the argument in class, discussing their difficulties. Questions 19 and 23 may be useful as preparation for question 24.

Demonstration

4.6 Longitudinal waves on a wave model made of trolleys and springs

In Unit 1 the Young modulus E for steel was related to a simplified atomic model of atoms spaced in a cubic array with spacing x and separated by springy bonds with spring constant k. (k is the force exerted by a stretched or compressed bond divided by the increase or decrease in the distance between the atoms.) The relation was $E = k/x$.

How might such an arrangement of atoms propagate a compression wave? Produce a row of trolleys linked by compression springs. See figure 15. Each trolley represents an atom, each spring the stiffness of a bond. Compression and expansion pulses can be sent along the row of trolleys.

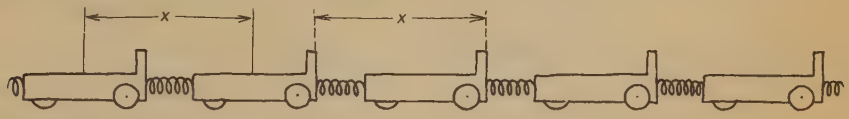


Figure 15
Trolleys linked by compression springs.

At this point, it is convenient to measure x, the distance between trolley centres, and k, the force needed to compress a spring-bond divided by the compression made. The results will be needed shortly, and are a reminder of the meaning of x and k meanwhile.

Theoretical argument for the speed of a wave

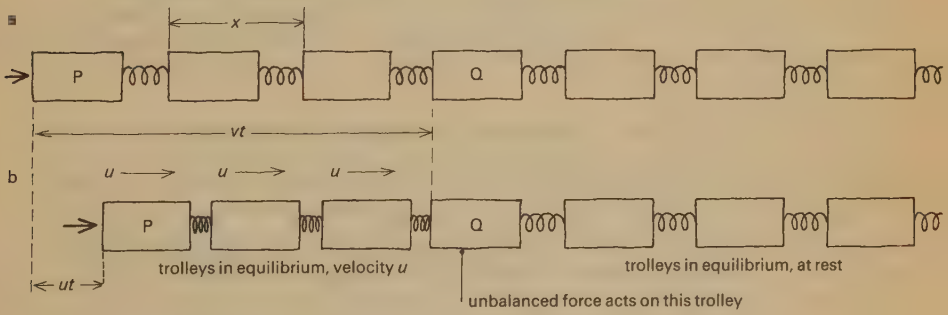


Figure 16
Wave on a row of trolleys.

Figure 16 shows a special sort of wave about which it is convenient to argue. In figure 16*a*, trolley P is being pushed by a steady force on its left. It accelerates up to velocity u. It then continues to move at this velocity, the force on its left being balanced by one from the compressed spring on its right. Figure 16*b* shows the wave after time t. The wave front, travelling at velocity v, has reached trolley Q,

Textbook

Sears and Zemansky, *College physics*, Chapter 21, gives an essentially similar argument.

Measuring the speed of the wave

The wave speed along the trolleys can be measured as a continuation of demonstration 4.6. The method suggested is intended to help students to understand the method used to measure the speed of sound in a metal rod in demonstration 4.7, and the two could well be done side by side.

A row of about four trolleys is sent, travelling as a whole, towards a rigid wall, as shown in figure 17. When the front trolley hits the wall, it stops. The others come up behind it and stop, one after the other, so that a compression wave front travels along the row to the rearmost trolley.

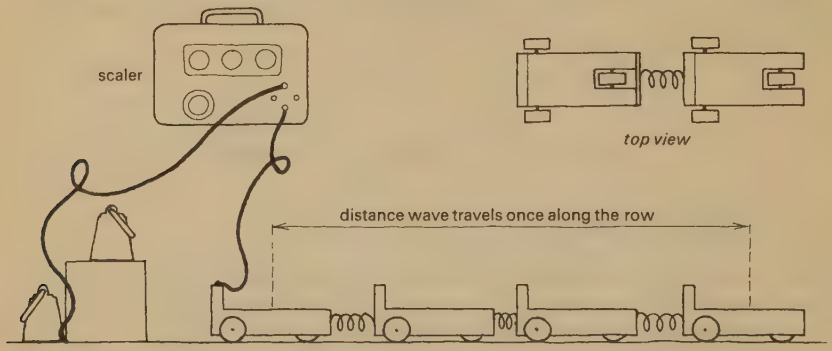


Figure 17
Timing a pulse up and down a row of trolleys.

This last trolley stops, and then moves away from the wall, followed one by one by the other trolleys. An 'expansion' wavefront, in which the trolley spacing returns to normal, travels along the row back to the front trolley. This trolley has meanwhile been pressed firmly against the wall by the compression in the spring attached to it. Finally, the 'expansion' wave front reaches the front trolley and it moves away from the wall.

The time the front trolley spends in contact with the wall, which is equal to the time needed for a wave to travel twice along the row of trolleys, is recorded using a scaler. The contact between trolley and wall is used to keep the scaler counting milliseconds whilst the contact is closed. The wave speed follows from the time and from the distance travelled by the wave.

The wall can be an aluminium block held firmly in place by several 1 kg masses piled on and behind it. Trouble will be experienced if the block can move. One lead to the scaler terminals which start the scaler counting when short circuited (green terminals) goes to the block. The other, which should be very flexible, goes to a contact on the front of the front trolley. It may be convenient to push a bare end under the metal strip found on the front of some types of trolley.

The trolleys must be set travelling towards the wall as a whole. This can be done by spreading a pair of hands over the whole row of trolleys and pushing all of them together.

It is convenient to repeat the impact several times without resetting the scaler, so that it records the total time for all the impacts; the average time for one impact is then calculated.

Notice that for calculating the speed, the length of the row is the distance from the middle of the first trolley to the middle of the last one (or, of course, from the front of the first to the front of the last). The wave travels twice this distance.

which is a distance vt ahead of where trolley P used to be. Trolley P, like the others between it and trolley Q, has been moving at steady velocity u and is a distance ut ahead of its previous position.

Before the wave passed, the trolleys were spaced at a distance x. By how much has the spacing x been compressed in the region between P and Q? The wave has travelled a distance vt, so passing over a number vt/x of trolley spacings. The total compression is ut, so the compression of each spacing is

$$\frac{ut}{vt/x} = \frac{u}{v}x.$$

As the wave front reaches a trolley like Q in figure 16*b*, all the springs behind are compressed, while all the springs ahead of Q are not. The net unbalanced force on such a trolley is $k\frac{u}{v}x$. As stated above, k is the spring constant for a pair of linked trolleys. The force is provided by the external force pushing on the left of trolley P. It is transmitted via the compressed springs to each trolley in turn.

In time t between figure 16*a* and *b* this force has acted upon the number vt/x of trolleys across which the wave has passed, giving each velocity u and momentum mu, m being the mass of the trolley. The total change of momentum is equal to the momentum given to each trolley multiplied by the number of trolleys involved.

But *external force* × *time* = *total change in momentum*

So $$\frac{kux}{v}t = mu\frac{vt}{x}$$

Whence $v^2 = x^2\frac{k}{m}$ or $v = x\sqrt{\frac{k}{m}}$ since u and t cancel.

Test of the wave speed prediction

The speed of a wave along the trolleys and springs model can be measured, and compared with the value calculated from the measured values of x, k, and m. For example, $x = 0.35$ m, $k = 50$ N m^{-1}, $m = 0.95$ kg give a predicted speed $v = 2.5$ m s^{-1}. Agreement should be quite good.

Prediction of the speed of sound in steel

If a plane wave travelled along a steel rod, so that atoms in a row lying along the wave moved like the trolleys in the model, while atoms to either side moved in unison, it would not matter whether or not the rows were connected with each other by springy bonds, and the wave speed should be the same as the speed along a row. See figure 19.

Macroscopic argument for the speed of sound in a solid

Teachers may wish to be able to show how the speed of sound can be calculated without any assumptions about atoms.

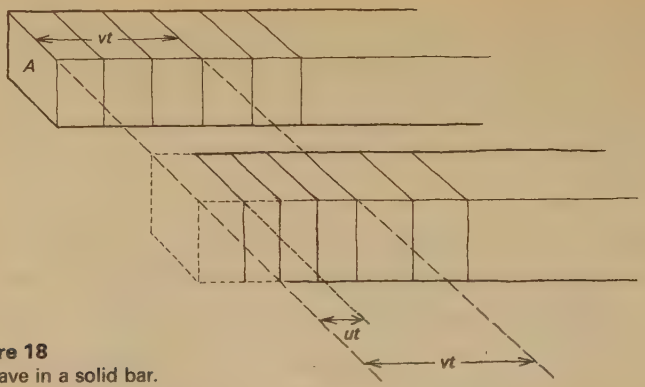


Figure 18
Compression wave in a solid bar.

As before, the end of a bar is imagined set in motion at velocity u. In time t the wave travels vt, so that a length vt of the bar has been compressed by an amount ut. If the bar has area A,

external force $F = EAut/vt = EAu/v$.

The mass set into motion at velocity u is the mass in length vt; that is, $vtA\rho$. It has momentum $uvtA\rho$. Since the force multiplied by the time is the change in momentum,

$$tEAu/v = uvtA\rho$$

whence $v^2 = E/\rho$.

See also page 55, for the speed of sound in a gas.

Students' book

Question 25 is about scaling the trolley wave speed (a few metres per second) to the speed of sound in steel (several thousand metres a second) by looking at how m, k, and x change in value.

Question 26 shows how the wave speed in a solid may be written in terms of the Young modulus and the density.

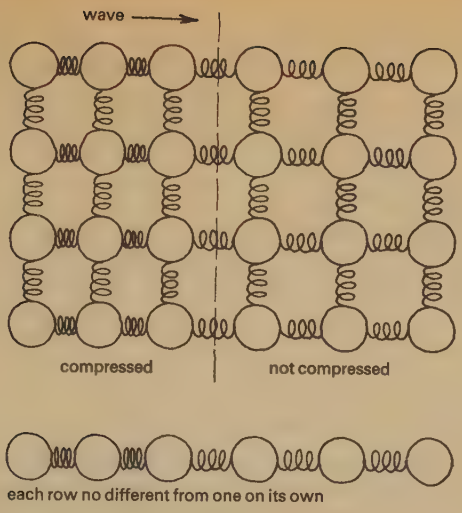


Figure 19
A plane wave in an array of atoms.

From Unit 1, for steel:

E (the Young modulus)	20×10^{10} N m^{-2}
x (atom spacing)	2.5×10^{-10} m
k (spring constant = Ex)	50 N m^{-1}
m (mass of atom)	9.3×10^{-26} kg
ρ (density)	7.8×10^{3} kg m^{-3}

(The mass m is obtained from the density ρ and the spacing, using $m = \rho x^3$, supposing the atoms to be in a cubic array.)

Then $v = x\sqrt{k/m}$ gives a speed close to 5000 m s^{-1}. But k and m were obtained from the Young modulus E and the density ρ, and x can be found from the density and the Avogadro constant. So the speed can be expressed entirely in terms of the large-scale quantities E and ρ, as follows:

$$v = x\sqrt{\frac{k}{m}} = x\sqrt{\frac{Ex}{\rho x^3}} = \sqrt{\frac{E}{\rho}}$$

It is important to say that the wave speed calculation does *not* depend on the special, simple (and incorrect) cubic arrangement that was imagined for a model of the propagation of the waves along rows of atoms. Indeed, it can be calculated without making any assumptions about atoms at all.

Besides consulting tables for the speed of sound in steel (5100 m s^{-1}) an experimental trial is worth while.

Demonstration

4.7 Speed of sound in a metal rod

64 oscilloscope

1009 signal generator

504 retort stand rod, 1 m long 2

503–5 retort stand base, rod, and boss

1053 rubber band, about 10 mm long 2

52K crocodile clip
or
1053 adhesive tape

183 loudspeaker

1000 leads

1055 hammer, club or claw head, at least 0.5 kg

See figure 20. One 1 m long rod is hung on rubber bands below another supported on a retort stand. The suspended rod is connected to the output of the oscillator; it is convenient to have a 4 mm hole drilled in this rod. The earthed oscillator output and oscilloscope input terminals should be joined.

Connect the hammer head by a short lead to the oscilloscope input. This lead should not touch anything else. Check that the oscilloscope trace shows little or no pick-up from the mains or from the oscillator, when the hammer is held in the hand, using the oscilloscope settings below. Insulate the hammer handle if necessary. A rubber handled, steel shafted, hammer is ideal.

Instrument settings:

Oscillator	frequency	25 kHz.
	output	6 V amplitude sine wave, high impedance.
Oscilloscope	input switch	a.c.
	brightness	maximum.
	trigger	automatic, positive going triggering.
	input sensitivity	5 V cm^{-1}.
	time base and X gain	100 μs cm^{-1}, calibrated positions.
	X shift	trace to start on the screen, at left.

Turn the stability control just so far anti-clockwise that the 25 kHz trace appears when the hammer is held in contact with the suspended rod, but vanishes when this contact is broken. This adjustment is critical.

Tap the end of the suspended rod smartly with the hammer, hitting it end on. A train of oscillations should appear, lasting as long as the contact is made, as indicated in figure 20. The train should be about 40 mm long on the screen. Repeat as often as is needed to note (or mark with a wax pencil on the screen) the start and finish of the train of oscillations allowed to pass from oscillator to oscilloscope through the contact between hammer and rod. This contact remains closed while a pulse travels up and down the rod, that is, along twice its length.

The time scale can be calibrated by holding the contact closed and counting the number of 25 kHz oscillations between the two marks on the screen. This method eliminates error from the rather variable speed of the time base near the start of the trace.

Typically, there may be ten oscillations, giving a time of 0.4 ms for the pulse to travel 2 m, a speed of 5000 m s^{-1}. Frequencies above 25 kHz can be used, but the trace is then fainter. At lower frequencies, the end of the train is hard to pick out.

Demonstration

4.7 Speed of sound in a metal rod

The speed of sound in a rod of steel or other metal can be measured by a method exactly like that used in experiment 4.6 to time the wave along a row of trolleys and springs.

One end of a steel rod is hit with a hammer. The hammer remains in contact with the rod while a compression pulse travels up the rod, successive layers of steel moving away from the hammer. An expansion pulse returns from the far end, again with layers of steel at the front of this pulse moving away from the hammer, until the pulse reaches the end that was struck, and the contact is broken.

If the predicted speed of about 5000 m s^{-1} is right, the 'bounce time' for a rod 1 m long will be about 0.4 ms. Most school scalers will not record so short a time. An oscilloscope can be used, employing either its calibrated time base, or using time markers from an oscillator running at a frequency such that there are at least ten oscillations within 0.4 ms, that is, at about 25 kHz.

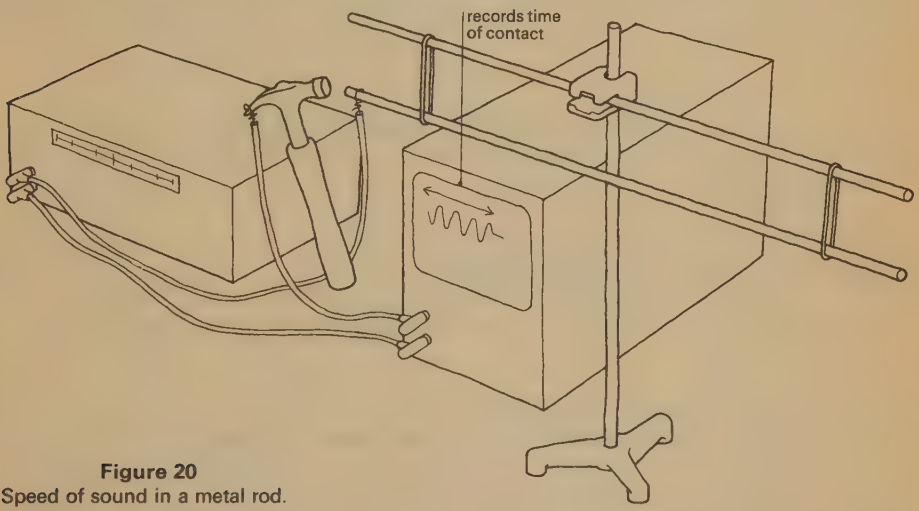


Figure 20
Speed of sound in a metal rod.

The 'bang and time' method proposed here and in the details opposite is not reliable to better than ten per cent, but can confirm that the theoretical prediction is about right. The accuracy can be improved somewhat by tuning the oscillator and a loudspeaker to the note emitted by the rod after it is struck. This calls for a short discussion of standing waves. A really accurate measurement, not worth attempting except as a private exercise for an interested student, could involve exciting the rod into continuous longitudinal oscillation and counting the oscillations with a high speed counter over a fixed interval of time. (The electronics kit might be suitable.)

Rods of other metals can be tried, and the speed of sound in them compared with the speed predicted from $v = \sqrt{E/\rho}$.

The hammer must not be such that the contact time is fixed by the time for a wave to travel across the hammer and back. Rod-shaped hammer heads are to be avoided, especially if they are not massive.

Frequency of longitudinal oscillations

Use the oscillator and a loudspeaker to produce a sound at about 2.5 kHz. Tune this note to be the same as the high pitched ringing sound emitted by the rod for several seconds after it is struck at one end. For this, clamp the rod at its centre in a boss head. Beats between the two notes can be heard and tuned out. The period of the oscillations is then equal to the time for a wave to travel twice along the rod.

Other sorts of mechanical wave

Students should be clear that the compression wave theory (*Students' book,* question 24, together with questions 25 and 26) is offered as an example of the possibility of making such calculations.

The brief discussion of other waves, centring around formulae for various wave speeds, is meant to remind students of the general application of the type of argument exemplified in the discussion of compression waves in solids. The formulae appear in the *Students' book,* page 112.

Students may wish to see how some of these other wave speed calculations go. They can be referred to the *Students' book,* questions 27 and 28. These optional questions discuss, respectively, the speed of transverse waves on strings, and the speed of gravity waves in shallow water.

Water waves

Formulae given opposite are for limiting cases only. In water whose depth is large compared to the wavelength, the wave speed expression contains two terms, one for surface tension effects and one for gravity effects. It is

$$v = \sqrt{\frac{g\lambda}{2\pi} + \frac{2\pi\sigma}{\lambda\rho}}$$

where the symbols have the meaning given opposite.

For short wavelength (ripples), the second term predominates, and the speed is approximately

$$v = \sqrt{\frac{2\pi\sigma}{\lambda\rho}}.$$

For long waves, the first term predominates, and the speed is approximately

$$v = \sqrt{\frac{g\lambda}{2\pi}}.$$

Interested students may be referred to Tricker, *Bores, breakers, waves and wakes* or Barber, *Water waves*. Bores are a special case of shallow water waves ($v = \sqrt{gh}$) and are the type of wave discussed in question 28. A bore can easily be made in the water trough (experiment 4.5d) by sweeping water along at a steady rate using a wide paddle.

The speed of other mechanical waves

In expressions like $v = x\sqrt{k/m}$ or $v = \sqrt{E/\rho}$, why *must* quantities like E and k, which give the size of the force needed to distort the material through which the wave passes, be in the numerator, so that v rises if E or k rise? (Because the larger the forces, the quicker the next bit of material ahead of a wave front will respond.) Why *must* m or ρ appear in the denominator, so that v falls if m or ρ increase? (The larger the mass to be given a certain speed by the wave front, the longer this will take.)

The speeds of many other waves can be calculated by arguments similar to those used for the speed of sound in a solid bar.

Speed of sound in a gas $= \sqrt{\gamma p/\rho}$

ρ is the density. γp is the elasticity, not this time of a 'rod' of air, but the pressure needed to produce a certain change in volume. The factor γ, varying from gas to gas, allows for the fact that the compressed gas gets hot, so that the pressure rises by a little extra on that account.

Speed of transverse waves on a spring or string $= \sqrt{T/\mu}$

T is the tension in the spring, and μ is the mass of each metre of it.

Speed of tiny ripples on water $\simeq \sqrt{2\pi\sigma/\lambda\rho}$ if wavelength is small

σ is the surface tension; ρ the density. These waves have a speed which depends also on the wavelength λ. Surface tension forces move these waves along; gravity forces on the tiny humps of water are there, but are too small to matter.

Speed of long waves in deep water $\simeq \sqrt{g\lambda/2\pi}$

g is the acceleration of gravity, for gravity is the force moving these waves along. Surface tension forces are now too small to matter. The density does not appear because if it rises, the force acting and the mass to be moved both increase by the same factor, with no net effect on the response time of water ahead of a wave front.

Speed of waves in shallow water $\simeq \sqrt{gh}$ ($\lambda \gg h$, amplitude $\ll h$)

h is simply the depth of the water.

Units of combinations of quantities

It is worth considering the units of the quantity v given by some of these expressions.

Consider the waves on a spring. The expression T/μ has units N/kg m^{-1}. Replacing the unit N with the more fundamental equivalent kg m s^{-2} gives kg m s^{-2}/(kg m^{-1}) = m^2 s^{-2} = (m s^{-1})^2, as it should do.

Experiment

4.8 Testing other wave speed expressions

1013 long spring

81 newton spring balance (10 N)

507 stopwatch or stopclock

501 metre rule

20 domestic balance (5 kg)

It is practicable for most students in a class to do this experiment at the same time. Whether it is desirable is up to the teacher.

Time a pulse sent along the spring over as many transits as possible. The mass per unit length is found by weighing the spring, and dividing the mass by the stretched length.

Shallow water waves

see 4.5d.

Ripples on water

90 ripple tank kit

1009 signal generator

1060 vibrator

501 metre rule

For ripple tank instructions see Nuffield O-level Physics, *Guide to experiments III*, experiment 4.

The shadow of a ripple is easily seen. The scale change produced by shadow projection can be found by putting an object of known size in the water and measuring its shadow.

For reasonable agreement with the simple ripple speed formula, the wavelength should be less than about 5 mm, when the speed is about 0.3 m s^{-1}. This is high for direct timing, and it will be best to measure the wavelength of waves produced by a vibrator fed at a known frequency of 60 Hz or more.

Sources of background information, pictures, etc.

Barber, *Water waves.*
Griffin, *Echoes of bats and men.*
Tricker, *Bores, breakers, waves and wakes.*

Bascom, 'Ocean waves'.
Bernstein, 'Tsunamis'.
Bullen, 'The interior of the Earth'.
Griffin, 'More about bat "radar"'.
Oliver, 'Long earthquake waves'.

The first three titles are books. The rest are reprints. Tricker is superb; Barber could be of great interest to a mathematically inclined student; Griffin is descriptive and inexpensive.

Experiment

4.8 Testing other wave speed expressions

As a demonstration of their practical skill, students may be inclined to test one or more of these speed expressions, or they may become interested enough to want to save one for an individual investigation. Since waves on springs were discussed earlier, it may be well to test the expression for speed of these, at least.

Waves, their interest and importance

It would be sad if the wide variety of waves mentioned above were represented merely as formulae, when there is so much of beauty and interest associated with them. It is suggested that teachers talk a little about one or two that interest them, using sources such as those opposite, or that they persuade a student to prepare a short illustrated talk.

Some possible lines to follow up include:

Sound ranging. Sonar and the sound navigation system used by bats.

Ultrasonics. Using this to detect flaws in railway lines, or in sterilizing liquids.

Tides and the flow of tidal waters in channels.

Bores in tidal rivers.

'Tidal waves' produced by undersea earthquakes ('tidal waves' are not tidal, and are more properly called tsunamis).

Wakes of boats.

Sea waves and breakers. Waves travel thousands of kilometres from ocean storms, the long wavelength waves arriving first. The distance of the storm can be deduced from the delay in arrival of waves of differing length.

Earthquakes and the compression and transverse waves they generate, which give information about the interior of the Earth.

Part Three

Mechanical oscillations

Time: two to three weeks

Exhibition

4.9 What is a clock?

4.9a *Oscilloscope with slow time base*

64 oscilloscope

Set the time base at 100 ms cm^{-1} and the stability control at maximum.

4.9b *A ball rolling on a curved track*

See Nuffield O-level Physics, *Guide to experiments V*, experiment 72h.

A board about 0.5 m square has two pieces of curtain rail attached to it, one piece being bent into an arc of a circle, the other being V-shaped or parabolic. The board is arranged vertically so that ball bearings or marbles can roll on the curtain rails.

4.9c *A rubber ball bouncing*

1053 rubber ball

The ball is allowed to bounce on a hard surface.

4.9d *A pendulum swinging*

10F set of parts for heavy pendulum

See Nuffield O-level Physics, *Guide to experiments I*, experiment 30b (broomstick pendulum).

4.9e *A scaler counting regular pulses*

1009 signal generator

130/1 scaler

1000 leads

The signal generator is set to give square waves at about 100 Hz and the high impedance output terminals are connected to the unpolarized scaler input. Whether or not the scaler counts depends on the generator output voltage. The output voltage should be slowly increased until the scaler counts regularly.

4.9f *A scaler counting slow, random counts from a GM tube*

130/1 scaler

130/3 GM tube holder

130/5 thin window GM tube

The apparatus is set up to show the slow random background counts from the GM tube.

Oscillations

Many sorts of wave involve rhythmic; repetitive, to-and-fro motions, or oscillations. But oscillations have an interest and importance that go beyond the connection with wave motion. The oscillations of car wheels and car bodies are of interest to the car user and are a problem for the designer, for example. The regular rhythm of oscillation has also something to do with another kind of problem: the meaning of measuring time.

Exhibition

4.9 What is a clock?

A selection of repetitive events, such as those produced by the following items, can be set up so that the class can look at all the events together.

- **a** Oscilloscope with slow repeating time base
- **b** A ball rolling on a curved track
- **c** A rubber ball bouncing
- **d** A pendulum swinging (perhaps the O-level broomstick pendulum)
- **e** A scaler counting regular pulses (at, say, 100 counts a second from an oscillator)
- **f** A scaler counting slow, random counts from a GM tube
- **g** A rotating motor-driven turntable (perhaps carrying a wire which clicks against a fixed card as it rotates)
- **h** A watch or clock ticking (near a microphone, connected to an amplifier and loudspeaker)
- **i** A slow-running multivibrator circuit, flashing a light regularly
- **j** A slow-running flashing neon circuit
- **k** Water dripping slowly from a long tube

Just looking at them, some events seem to repeat regularly, some not. Several interesting questions arise. How could one tell if one event repeated regularly? Use a clock? How would one know that the clock ticked off equal time intervals? What would be observed if pairs of these repetitive events were compared with each other for rate and for regularity?

Which of these events count as clocks? Which are good clocks? As the discussion develops, deeper questions may arise. What is time? Does time run steadily?

Students might use clocks at home to test their pulse rates for regularity. (Galileo used his pulse to test a pendulum, or so the story goes.) Does irregularity matter – is it possible to use radioactive decay as a clock? (Some may have heard of radiocarbon dating.) Reference can be made to modern time standards and to astronomical methods of time measuring.

It is amusing to ask 'Could time run backwards?', 'Would we know if it was doing so?'. Such questions may arise again in the second year work (Unit 9).

4.9g *A rotating motor-driven turntable*

154/1 turntable

150 fractional horse power motor, with gearbox,
and
59 l.t. variable voltage supply,
or
134/1 motor-driven stroboscope

1053 card, adhesive tape

1054 wire

1000 leads

A short piece of wire is taped to the turntable and the card is fixed so that the wire just touches it as the turntable rotates.

4.9h *A watch or clock ticking*

507 stopwatch or stopclock

157 microphone

181 general purpose amplifier

183 loudspeaker (if not already fitted to the above)

The apparatus is set up to amplify the ticks from the clock.

4.9i *A slow running multivibrator circuit flashing a light regularly*

1075 electronics kit

1033 cell holder with 4 U2 cells

1000 leads

The connections are shown in figure 21.

4.9j *A slow running flashing neon circuit*

15 h.t. power supply

1017 resistance substitution box, 1 MΩ

1051 capacitor, 1 μF 500 V

1040 clip component holder *2*

92S/T neon lamp and m.e.s. holder

1000 leads

The circuit is shown in figure 22.

k *Dripping water*

1055 burette, stand, and beaker

Let the water drip slowly from the burette so that the rate is roughly steady over short times, but decreases over long times.

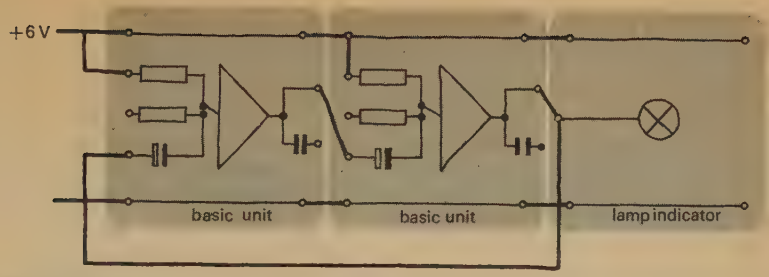


Figure 21
Multivibrator using electronics kit.

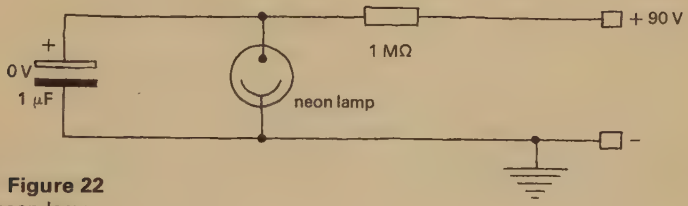


Figure 22
Flashing neon lamp.

Alternative discussion : chronometers

In the middle of the eighteenth century, John Harrison solved the problem of making a clock that would keep good enough time to navigate a ship by, despite the rolling of the ship in the sea and despite changes in temperature. Students impatient with philosophizing may prefer to consider this practical matter.

It will be necessary to explain that a clock is used in navigation to find the longitude, that is, one's position east or west of a given place. Crudely, if sunrise occurs six hours late or early, one has travelled a quarter of the way round the Earth. An error of one minute in time makes a navigational error of nearly thirty kilometres at the equator. Between 18 November 1761 and 21 January 1762 one of Harrison's clocks was taken on a voyage from England to Jamaica. On arrival, it was tested and found to be in error by only five seconds. How could such a test be made, there being no other clock that could be taken along for comparison? (By finding the longitude of Jamaica by other, astronomical, means and noting the time of, say, noon in Jamaica.)

Purpose of the discussion of time

Many sixth form students are ready for a little philosophy, though it would be wrong to labour the discussion. The questions, 'What is time?', 'Does time run evenly?', 'Could we know if it did?' are deep and important. At present, such arguments exercise physicists when they think about fundamental particles, and in the past influenced both Newton and Einstein. They are practical questions, too. Not so long ago, clocks were carried about the country to compare local times when the coming of railways made it useful to have a uniform Railway Time for the whole country. At present, accurate 'atomic' clocks in different continents are compared, and there are sometimes discrepancies that are not easy to explain.

We advise letting the class make the running in discussion, and cutting it short when they grow tired of it. It will be enough if they finish by thinking that these matters may be important, without having resolved them.

Reading

One or two of the following sources may provide useful reading.

Butler and Messel (eds), *Time*, especially the chapter 'The arrow of time', Bronowski.
Hurley, *How old is the Earth?*
Project Physics, *Reader Unit* 3 'The arrow of time', Bronowski.
Rogers, *Physics for the inquiring mind*, page 339.

Deevey, 'Radiocarbon dating' (reprint).
Lyons, 'Atomic clocks' (reprint).

For teachers:
Essen, 'The measurement of time and frequency' in Contemporary Physics, *Sources of physics teaching*, Part 2.
Feather, *Mass, length and time.*
Feynman *et al.*, *The Feynman lectures on physics*, Volume I, Chapter 5.
Gould, 'John Harrison and his timekeepers'.

Students' book

Questions 29 to 32 are about clocks and time measurement.

Practical problems involving oscillations

Illustrative material can be drawn from:
Students' book, the article on 'The Severn bridge'.
Bishop, *Vibration.* (Many examples and some good photographs.)
Frischmann, 'Tall buildings' (reprint). See page 128.
Appendix A to this *Guide* (effects of vibrations on man).

Film loops

'Tacoma Narrows Bridge collapse.'
'Wind-induced oscillations.'

Students' book

Question 33 is about the practical need to understand oscillations.

The captain of HMS Centurion, which carried Harrison on an earlier trial in 1736, wrote of him,

> '. . . the instrument is placed in my cabin, for giving the man all the advantage that is possible for making his observations, and I find him to be a very sober, a very industrious, and withal a very modest man, so that my good wishes can't but attend him; but the difficulty of measuring time truly, where so many unequal shocks and motions, stand in opposition to it, gives me concern for the honest man, and makes me feel he has attempted impossibilities.'
>
> *From Short, J. (1763)* An account of the proceedings in order to the discovery of the longitude: at sea; relating principally to the time-piece of Mr. John Harrison; *T. and J. W. Pasham quoted in Gould,* John Harrison and his timekeepers.

The practical importance of oscillations

Many engineers and designers spend a good deal of time producing and controlling oscillations. These may be the oscillations of a record player pick-up, of air in a loudspeaker cabinet, or of parts of machinery like the shuttle of a loom. Many other engineers spend time preventing oscillations, whether of a suspension bridge, of a building or chimney subject to earthquake or to wind, or of the interior of a car.

Oscillations vary in their nature and in their causes, and they can only be used or controlled if these factors can be understood. Fortunately, despite their variety, many oscillations have much in common, and ideas which help understanding of one also help for others. This Part is concerned mainly with these generally useful ideas. In the next experiment, one important class of oscillators – the 'harmonic' oscillators – are identified by their common behaviour.

Group of experiments

4.10 The motion of oscillators

Aim

These experiments aim at giving students experience of several kinds of oscillation, so that they may see that some of them share enough properties – time nearly independent of amplitude and sinusoidal 'time trace' – to make it worth grouping them together. The abstract idea of a 'harmonic oscillator' is thus illustrated by pointing at examples. Closer definition comes later, when the idea has been clarified further.

We suggest that each group of students look in detail at no more than two or three samples, while being encouraged to see what others are doing. Results could be reported informally in the course of general discussion, as they will probably be too tenuous for more formal treatment.

Students will require stopwatches and should have access to a variety of other materials and equipment. It may be necessary for them to request what they want after an initial period of experimenting.

A choice should be made from the following. Some which are clearly not regular should be included.

Practical details

For experiments 4.10a to j, see Nuffield O-level Physics *Guide to experiments V*, pages 120 to 127, as follows.

- **a** *Pendulum* experiment 73
- **b** *Torsion pendulum* experiment 72b
- **c** *Lath with load* see below, and experiment 72c
- **d** *A large model of a watch balance wheel* experiment 72d and page 296 (Appendix V)
- **e** *U-tube containing liquid* experiment 72e
- **f** *Ball in a bowl* experiment 72f
- **g** *Inertia balance* experiment 72g
- **h** *Ball rolling in a curved track* experiment 72h
- **i** *Mass hanging on a spring* experiment 71b
- **j** *Trolley tethered between springs* experiment 71c or Advanced Physics experiment 4.11

4.10c	*Lath with load* (large-scale version suitable for demonstration)
1053	long lath (2.5 m by 75 mm by 10 mm)
55	friction kit
150	fractional horsepower motor
9F	lineshaft unit
59	l.t. variable voltage supply
44/1	G-clamps (large) *2*
32	1 kg mass *2*
1000	leads

Also required: a fine pointed felt tip or ball point pen; clean smooth paper about 0.5 m by 0.2 m; layers of soft paper such as kitchen paper towelling.

Figure 23 shows an arrangement which can be used to show an exponential decay of oscillation amplitude. Note that such a decay arises when the damping force is proportional to velocity and will not always or necessarily appear in other cases.

Group of experiments

4.10 The motion of oscillators

In the earlier demonstrations students should have asked which 'clocks' kept regular time, and they can now be asked to look carefully at a range of oscillators, and see whether each of these does so or not. Some will oscillate at a steady rate – judged by the oscillating balance wheel in a watch – while others will not.

They may also be asked to find out how the oscillators move. A quick demonstration with, say, a pendulum with an ink brush, can show how the motion traces out a wavy line. Then students can see whether this fingerprint, the to-and-fro line dying away, appears in any other cases. For some, they will have to use ingenuity to tease the time trace out of the apparatus. They should also be asked what happens to the energy in the oscillations.

A choice can be made from the following, of which **a** to **j** appear in the Nuffield O-level course, year V.

- **a** Pendulum. A means of obtaining a trace of the motion should be available, and the pendulum can be used as a demonstration to start the work off
- **b** Torsion pendulum
- **c** Lath with load
- **d** A large model of a watch balance wheel
- **e** U-tube containing liquid
- **f** Ball in a bowl
- **g** Inertia balance (wig-wag)
- **h** Ball rolling in a curved track (circular, parabolic, vee-shaped)
- **i** Mass hanging on a spring
- **j** Trolley tethered between springs
- **k** Bar magnet suspended over another magnet
- **l** Air track vehicle running freely between elastic barriers

At least one of the last two, or a similar substitute, should be included so that a motion which is not harmonic is represented.

A large-scale version of **c** can be used in a demonstration to draw students' attention to the damping of the oscillations. The vibrating lath can be arranged to give a beautiful exponential decay of oscillation amplitude, and students should be asked if they can recognize the shape of the envelope of the oscillation curve from earlier work ('The exponential decay of charge on a capacitor' in Unit 2).

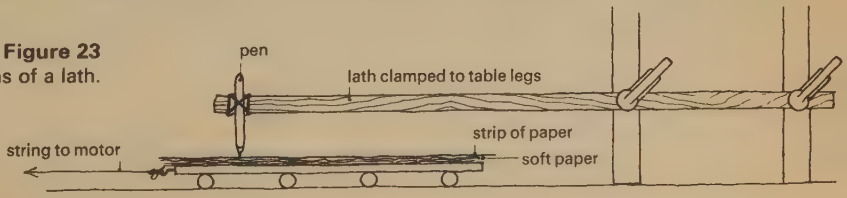


Figure 23
Oscillations of a lath.

Clamp the lath firmly at one end to a pair of table legs, about 0.1 m above the floor. Masses can be attached to its free end with rubber bands or string. To obtain a time trace, make a pen on the end of the lath write on a moving paper strip. The paper can rest over several layers of soft paper on a wooden board running on steel rollers. The soft paper allows the pen to write smoothly despite bumpy movement of the moving board, and is needed if the decay of the oscillations is to be anything like exponential.

One way of towing the board and paper is to pull it along with a string wrapped round a shaft turned by the motor.

4.10k *Bar magnet suspended over another magnet*

50/1 cylindrical magnet

50/2 horseshoe magnet

503–6 retort stand base, rod, boss, and clamp

1053 nylon fishing line

Hang the bar magnet on nylon line or cotton so that it is horizontal and lies just over the poles of the horseshoe magnet resting on its back. A small piece of mirror and a suitable lamp should be to hand so that the oscillations can be observed by optical means.

4.10l *Air track vehicle running freely between elastic barriers*

1019 air track

1020 air blower

Improvise an arrangement for holding two tightly stretched rubber bands across the track about 0.5 m apart, so that the vehicle rebounds between them with little loss of energy. A system of magnetic repulsion can also be arranged, but with greater difficulty.

Students' book

Question 34 is about time traces.

Other damping demonstrations

See Unit 6, *Electronics and reactive circuits,* experiment 6.16, for an example of damped electrical oscillations. The loop 'Measurement of *"G"* shows speeded-up film of damped torsional oscillations.

Mathematics and physics

One major purpose of the following work on the harmonic oscillator is to illustrate the possibility and power of mathematical model building in physics.

We choose for an example of a mathematical model one which will have a wide variety of later uses, some within the course and many at later stages of education or in practical tasks. In addition, the model of the harmonic oscillator develops further the ideas about rates of change and their representation by derivatives, first considered in Unit 2 (in 'Decay of charge on a capacitor').

Of course all the oscillations tend to die away, their energy being spread out into their surroundings. Because the decay can be made slow, though not prevented, there is a useful abstraction to be made. This imagines there is no decay, so that the oscillations go on for ever. Though unrealizable, it makes some sense because many of these oscillators have a period of oscillation that does not depend on the amplitude, so that amplitude and period may be considered independently in studying the motion.

Students will find that they can alter the period of some oscillations. They may be encouraged to try simple experiments, adding masses and springs to see qualitatively what happens, but need to be brought back to the questions, 'What is the motion's time trace?'; 'Is the time independent of amplitude?'. The terms 'period' and 'amplitude' can be introduced in discussion.

Discussion of the results, when each group has tried one or two oscillators, will bring out that many keep steady time, while some do not. Those that do keep it tend to have a characteristic wavy trace, while those that do not may be different. The term 'harmonic oscillator' may be introduced now or, better, when the linear relation between restoring force and displacement has been discussed in what follows.

Analysing the motion of oscillators

Much of the rest of this Part will now be devoted to an analysis of the motion of one type of oscillator, using a blend of mathematics and experiment.

Physicists use mathematics a great deal, and the following work serves as an example of that use. It is a worthwhile example because the mathematical analysis of simple oscillators turns out to be rather widely applicable within physics and engineering. No apology need be made if this work seems to a student to be more like 'mathematics' than 'physics': the two are not as distinct as they may seem.

The method proposed is a development of that used in Unit 2; that is, numerical analysis of the differential equation, with solutions being drawn out graphically.

Students should realize, from comments before and during the teaching, that the work is meant to help them to understand two things. First, they should understand the idea of a rate of change and the way in which it is handled in a mathematical model. Second, they should understand the value of mathematical models in physical inquiry. The latter means learning to be able to think of other cases where a mathematical model might be useful, and being able to think out what kind of use it might have.

It must, then, be clear that although the work on the harmonic oscillator has its intrinsic interest and importance, both it and the particular teaching method adopted are to be seen as examples of a type of thinking that is important in physics rather than as things to be learned for their own sakes.

Students' book

Questions 36 and 37 develop the arguments about the constant time of an oscillator. They should be easy enough to be done without prior discussion, so that class time can be saved for discussion of the outcome, though any difficulties will need to be cleared up as well. Only one need be done. Alternatively, a similar argument is developed in Rogers, *Physics for the inquiring mind*, page 171.

A special kind of argument

It is not easy to know whether the kind of semi-quantitative argument, exemplified by those opposite about how oscillator time is independent of amplitude, really suits beginners. It may be that they please the expert more than they impress the beginner, for the expert can see how they cut through the formalism of algebra to the essence of the ideas behind it, while the beginner has yet to find out what ideas are of the essence.

Nevertheless, they seem worth a trial. Arguments like this are used by physicists and engineers during the first stages of considering the feasibility of a new proposal, or in estimating orders of magnitude. For instance, the fact that scaling up a load-bearing structure (whether an animal's leg or a bridge component) will raise the dead weight by the cube of the linear dimensions, but the strength by only the square of these dimensions, lies at the heart of the limitations imposed by existing materials on the size that things may be.

Another example comes from Laithwaite, who argues in *Propulsion without wheels*, that bigger electromagnetic machines are usually better machines just because an increase in size reduces both the resistance and the reluctance within the machine. (Both increase in proportion to circuit length, but decrease in proportion to circuit cross-section, so the net effect of an increase in scale is a decrease in both.)

It may be, however, that teachers will find that such arguments go best at the end of the mathematical development rather than at the beginning. We do not know the answer, if 'answer' there is. Here, as elsewhere, teachers will have to experiment and to use their judgment.

Demonstration and discussion

4.11 What factors determine the period of an oscillator?

This experiment provides a good opportunity to encourage simple commonsense reasoning, where students can use their dynamics in a slightly novel situation. They may need to have it made clear that such thinking is valued in this course.

One or two of the 'regular' oscillators from 4.10, in which mass and restoring force can be altered conveniently, are selected.

Why might an oscillator keep steady time?

There are simple arguments relating the steady time-keeping of an oscillator to the proportionality of displacement and restoring force. They are most suitable for study at home, in the form of questions from the *Students' book*. There are two equivalent versions. Both consider the time for a quarter of one oscillation.

1 The time is constant. If the amplitude is doubled, the average speed must be doubled. The double speed is acquired in the same time, so the average acceleration is doubled. Double the acceleration means double the force. So the average acceleration is proportional to the amplitude. This can be achieved by having the restoring force proportional to the displacement.

2 If the restoring force is proportional to the displacement, twice the displacement gives twice as much force, giving twice as much acceleration. Thus the velocity gained in any given short time is doubled, and twice the distance is covered in this same time. But the displacement was doubled to start with. There is just twice as far to go, and the double distance will be covered in the same time as the original motion.

An oscillator for which the restoring force was truly proportional to the displacement would be called *harmonic*. Few real oscillators behave so, except over a limited range. (There are other oscillators – like electrical ones – where the 'restoring force' can become a potential difference and the 'displacement' a charge.)

Demonstration and discussion

4.11 What factors determine the period of an oscillator?

Produce one or two of the harmonic oscillators, choosing ones whose mass or restoring force can easily be varied. On their previous experience, the class should be able to suggest how the oscillations might be speeded up or slowed down. Ask them to guess what change to, say, the mass would double the time of oscillation.

With those where the mass can be easily and obviously doubled, and where the restoring force can be doubled, try some simple tests as they are suggested. Is the time halved, doubled, or what? Simple quick measurements with stopwatches are called for.

Try to arrange at least once to double the force and the mass together and get the same time back again. Questions based on the following argument may usefully be combined with this investigation.

1 (from 4.10i)

2A expendable steel spring *4*

1053 rubber band *4*

31/2 weight hanger with slotted weights (100 g) *2*

503–6 retort stand base, rod, boss, and clamp

507 stopwatch or stopclock

A suitable mass should be suspended from the rod part of the clamp by means of a spring or rubber bands. A suitable mass for the spring is 400 g. The time for a known number of oscillations should be measured and the effect of changing first the force constant and then the oscillating mass should be found. Connecting springs (or rubber bands) to the mass in series decreases the force constant whereas connecting them in parallel increases it. The period should also be measured with double the original values of force constant and mass.

2 (from 4.10j)

106/1 dynamics trolley *4*

2A expendable steel spring *8*
or
1053 rubber band *8*

503–4 retort stand base, and rod *2*

44/1 G-clamps (large) *2*

107 runway for trolley

Figure 24 shows a convenient arrangement. Each end of the trolley is tethered by two or three springs in series to allow a reasonable amplitude. Further sets of two or three springs are added in parallel at each end to raise the force constant. To raise the mass, trolleys are stacked up on the tethered one, or other suitable masses are placed upon it. In this way the force constant and the mass can easily be changed by factors of two or four.

Students' book

Question 38 is about the effect on the time of oscillation of changing mass and spring constant.

Experiment

4.12 Detailed study of the motion of one oscillator

Trolleys may be used for individual experimenting, unless students are tired of them. If trolleys are used, the maximum velocity must not be too high, or the ticker timer records tend to be unreliable (see below). A photograph taken in a demonstration may well be better, especially if prints can be handed out.

106/1 dynamics trolley

503–4 retort stand base and rod *2*

44/1 G-clamp (large) *2*

107 runway for trolley

2A expendable steel spring *6*

1053 rubber band *6*

81 newton spring balance (10 N)

PHYSICS

Discussion of the effect of changing force and mass

Suppose, by changing the mass of the moving object or the stiffness of the springs, the oscillation time T were halved. Then (for the same amplitude) speeds must be doubled. But double speeds have to be reached in half the time, so accelerations must have quadrupled. How may the acceleration, and so the force, be quadrupled by altering the springs? The mass? (Four times stiffer; reduce to one quarter, respectively.) So a good guess is:

$$T^2 \propto m,$$

$$T^2 \propto \frac{1}{\text{forces acting}}.$$

The apparatus used for demonstration 4.11 will serve for a rapid test, doubling or quadrupling the mass, and doubling or halving the spring strength by adding or removing springs. It should be clear that the force that counts is the net restoring force, decided by the stiffness of the springs.

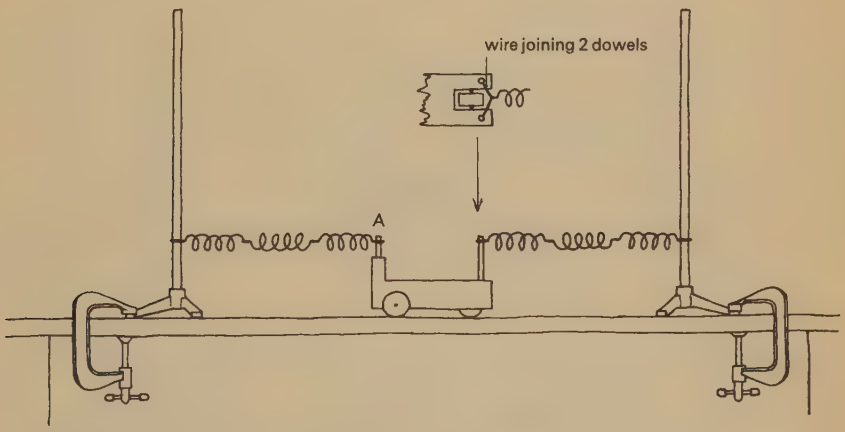


Figure 24
Trolley tethered between springs.

Experiment

4.12 Detailed study of the motion of one oscillator

So far, the discussion has been rather general. Harmonic oscillators have a 'wavy time trace'; this experiment seeks to record its exact form for one oscillator. Then a mathematical argument will show why the trace has this form, and find out whether it is related to known mathematical functions. In the process, the oscillation time T will be related to the mass m and the spring constant k, so these had better be measured.

108	tickertape vibrator, carbon paper disc, and tickertape
27	transformer
1000	leads
501	metre rule
	selection of known masses and Plasticine

For a photographic record, replace items 108 and 27 by:

133	camera (with stand and cable release)
171	photographic accessories kit
1054	film, monobath developer, printing paper P153, paper developer, and fixer
134/1	motor-driven stroboscope
	slide projector

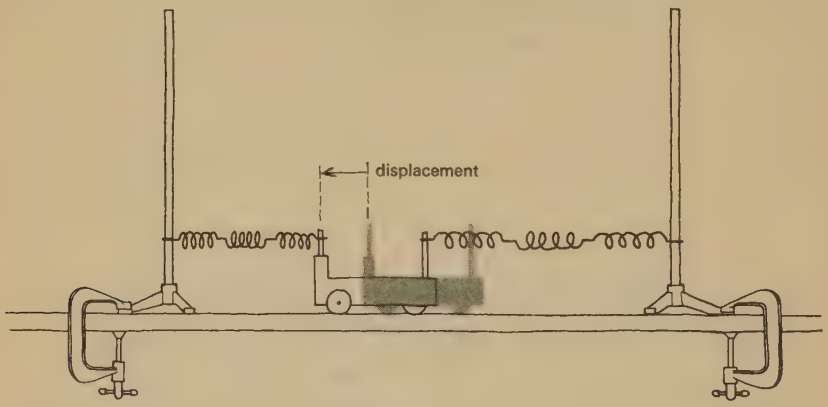


Figure 25
Trolley tethered between springs.

Figure 25 shows the arrangement, which is the same as that suggested for experiment 4.11, **2**.

If a photographic record is required, one peg of the trolley needs to be covered with shiny foil or painted white. Place a scale over the trolley.

Three springs (item 2A) in series at each end of the trolley allow a large enough amplitude and produce a convenient force constant. The force constant is measured by pulling the trolley aside by a measured distance using a spring balance. 20 N m^{-1} is typical.

The mass required to give the ratio k/m a simple value, such as 10 s^{-2}, is then calculated and the necessary mass added to the trolley. It is convenient for k/m to have a simple numerical value, so that the numerical analysis that follows goes more smoothly. The value suggested leads to a periodic time of close to 2 s.

The trolley runway should be tilted to compensate for friction, with ticker tape in place if it is used, but without the springs. Naturally, the friction compensation is limited to half an oscillation.

It will turn out to be convenient to have the ratio k/m a simple number, such as 10. (The unit is N m^{-1} kg^{-1}, which some students might show is identical to s^{-2}.) It is a good small exercise in dynamical measurement and calculation for students to measure k and calculate the mass that must be added to the trolley used in the experiment to achieve a certain value of k/m.

It is now possible to record accurate displacement and time data for a trolley tethered between springs, or for some alternative simple system. The data need only cover one quarter or one half of an oscillation. A graph of displacement against time should be plotted, with displacement taken to be zero at the centre of the oscillation, as in figures 25 and 26.

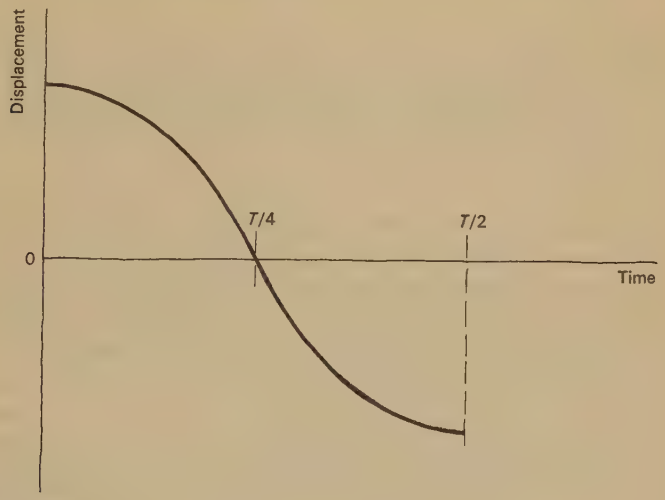


Figure 26
Displacement-time graph.

Keep a record of the time $T/4$ for the oscillator to reach one quarter of the way through a complete oscillation.

The next stage is to build up theoretical tools capable of showing the link between k, m, and T, and of explaining the shape of the displacement–time curve.

Students' book

Question 39 discusses displacement–time curves. Questions 40 and 41 go into greater detail.

Textbooks

Here, or during the following theory, students can be referred to textbooks, especially when they feel in need of security. Try:
PSSC, *College physics* (for dynamics).
PSSC, *Physics*, 2nd edition (for dynamics).
Rogers, *Physics for the inquiring mind*, Chapter 10.

Mathematically inclined students may like:
Feynman *et al. The Feynman lectures on physics*, Volume I, Chapters 21, 22.

Practical work during the theory

The section that follows is theoretical, though students will be actively employed. If practical work is needed, the investigation of the 'hacksaw blade' oscillator, experiment 4.13, could be begun now. This would help to ensure that students have adequate time for individual investigation.

The minus sign

The minus sign in $a = -\frac{k}{m}s$ has physical significance. The equation with a plus sign would describe some super-rocket, accelerating away faster and faster as it became more and more distant. For the oscillator, the acceleration is larger at larger displacements, but acts so as to reduce, not increase the displacement. For physical arguments, commonsense can sometimes replace correct signs, but mathematics is like a brilliant robot: it must be told *everything*. (If you tell a computer to search its memory for an item that happens not to be there it may go on and on searching for ever if you don't also tell it to stop looking if the item cannot be found.)

Reasons for taking constant acceleration first

As indicated opposite, students should be told where they are going, and why, so far as is practicable. The technical reason for starting with constant acceleration is that the equation $a = (constant)$ is both a first order differential equation $dv/dt = (constant)$ and a second order equation $d^2s/dt^2 = (constant)$. The first order version can be handled by methods as easy as those used for capacitor discharge in Unit 2. Then the way to handle a second order equation can be noticed by inspection of the way the solution develops. In particular, it becomes clear that acceleration is to be represented by the curviness of the graph – its rate of change of slope.

In practice, it is also useful to start with a problem about which students already know a good deal, so that mathematical difficulties are not compounded with difficulties of knowing what is going on physically.

The detour via constant acceleration stretches a long way in this *Guide*, from here to page 83. It should not take so long to teach as it may seem. Students need not have 'mastered' the ideas before they can go on; indeed it is in using them later that mastery will probably come.

An equation to solve

Since force F is proportional to displacement s,

$$F = -ks \qquad (k = \text{stiffness of spring, unit N m}^{-1}).$$

The minus sign indicates pull towards the centre, that is, in the opposite sense to s.

The acceleration a of a mass m is thus:

$$a = -\frac{k}{m}s.$$

For those who have calculus to hand, the equation may be written

$$\frac{d^2s}{dt^2} = -\frac{k}{m}s.$$

What does it mean, to say that this equation describes the motion? The equation says that the acceleration at some point is so much, so the trolley will pick up so much speed and travel so far. Then the acceleration is different, and new speed increases and distances moved have to be found. But with patience, it should be possible to find out how far the trolley will move, in a certain time, just using the acceleration recipe.

Preliminary discussion : analysing accelerated motion

In Unit 2 a graphical solution for the change of charge on a capacitor was drawn out, using the recipe $\Delta Q/\Delta t = -(constant)\,Q$. To find out how to do a similar job for the recipe $a = -(k/m)s$ it is necessary to find a way of handling accelerations graphically. But the acceleration is continually changing, and the easier case of constant acceleration is a better place to start. It is a by-way that will bring dividends.

Constant acceleration

The old problem of a falling ball, accelerating at close to 10 m s^{-2}, can quickly be given a graphical solution. Knowing that the distance fallen is given by $s = \frac{1}{2}at^2$ also makes it possible to check the graphical technique.

Suppose the ball starts at rest, at time $t = 0$. Over a short time, say 0.1 second, around $t = 0$ the velocity is pretty well zero, the distance travelled is also nearly zero, and the first bit of the graph must be flat, like the segment AB in figure 28.

But in the following interval of another 0.1 second, around the 'time-of-day' $t = 0.1$ second the average velocity will equal that at that time. If the acceleration is 10 metres per second each second, the velocity is 1.0 metre per second. So the next segment of the graph, BC, in figure 28, rises or slopes up at 1.0 metre per second, rising in all 0.1 metre in an interval of 0.1 second.

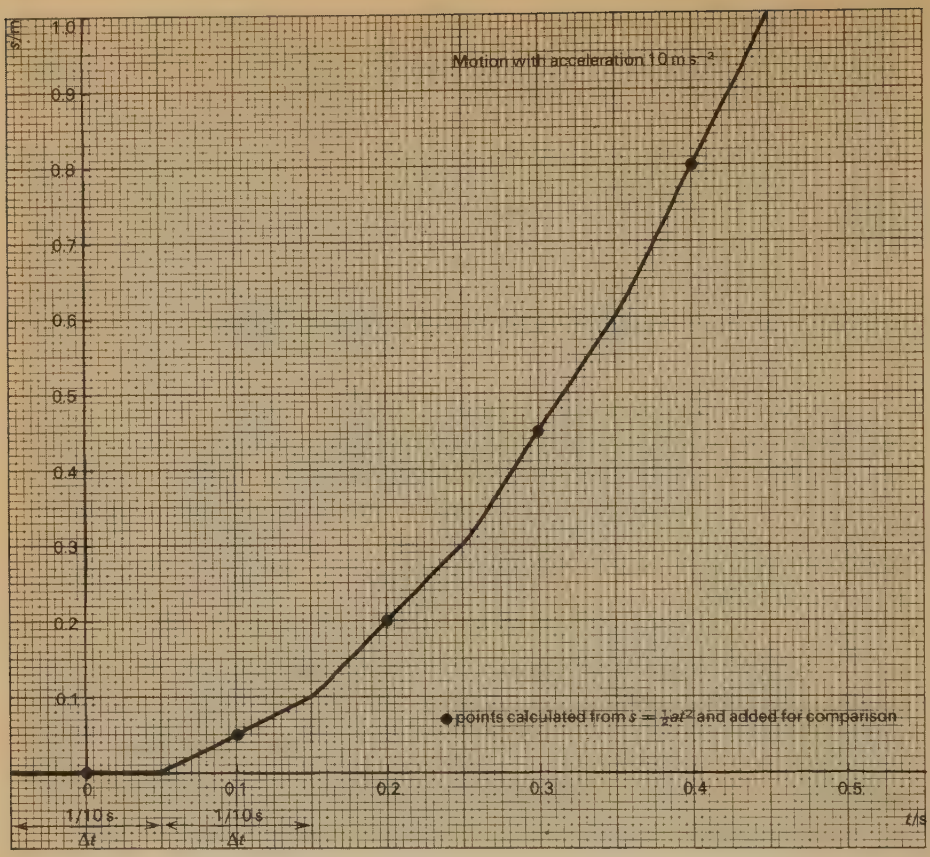


Figure 27

Boundary conditions and solutions of differential equations

The solution of the simplest sort of equation is a number. That produced here is not a number, but is a curve or a function. The term 'solution' is now being used in a wider sense than students are used to, and they may need help in seeing the difference.

Teachers will note that in general, the solution of a differential equation is a whole family of functions. Which one function is the solution of a particular problem depends on the boundary conditions. Here the boundary conditions are $s = 0$ and $v = 0$ at $t = 0$. Two conditions are needed to pin down one function as a solution of a second order equation. Only one is needed to do the same for a first order equation. (For example, setting $Q = 5 \times 10^{-3}$ C at $t = 0$ in $dQ/dt = -Q/10$.)

In all the numerical solutions proposed, the boundary conditions are built into the problem and the point needs no formal discussion beyond the admission that the solution obtained is, of course, a special case.

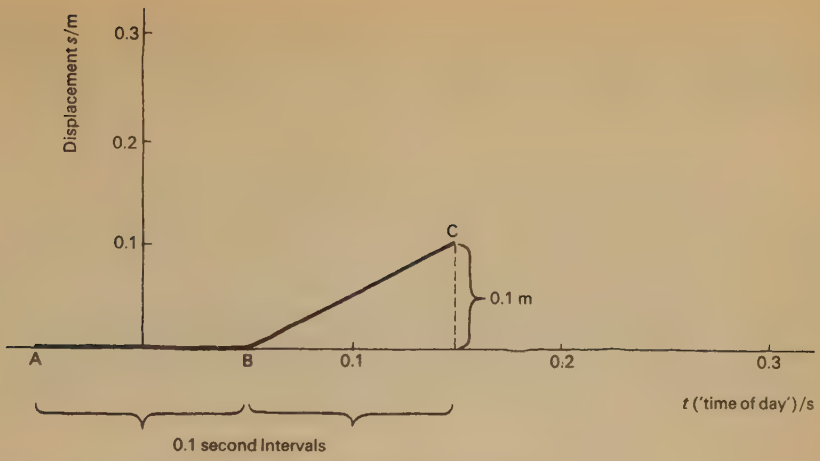


Figure 28
Beginning a displacement-time graph for constant acceleration.

The next 0.1 second interval centres around the time 0.2 second from the start, when the velocity will be averaging $10 \times 0.2 = 2$ metres per second. So the next section of the graph slopes up twice as steeply, rising 0.2 metre over 0.1 second. And so we go on. The rule is easy: around each time t draw a section of line for velocity $10t$ which must rise an extra distance (0.1) $10t$ metre.

Note that for successive equal time intervals Δt, the velocity rises by the *same amount each time*. The acceleration is constant.

Such a graph is shown in figure 27. The circles mark points calculated from $s = \frac{1}{2}at^2$, for comparison. It predicts, for instance, that a heavy ball will fall 1 metre in 0.45 second, and this could be tested using the O-level arrangement with a scaler to time the fall, if the class feels the need. Or a ball could simply be dropped, and the class be asked to estimate the time.

The equation $s = \frac{1}{2}at^2$ and the kinked 'curve' are both 'solutions' of the equation:

$$\text{acceleration} = 10 \text{ m s}^{-2}$$

or, better, of

$$d^2s/dt^2 = a \qquad \text{where } a = 10 \text{ m s}^{-2}.$$

The graph is an *approximate* solution: it is near to the exact solution, and although where it is wrong it always makes the distance come out a little too small, the graph does not drift off course (that is, the errors do not accumulate).

Revision of dynamics

The *Students' book*, questions 35, 40, and 42, can offer some practice in dynamical thinking. If experimental work is needed, see Nuffield O-level Physics, *Teachers' guide IV*, and *Guide to experiments IV*, experiments 1 to 12.

Application of approximate methods of calculation

Some students may feel that approximate methods are a cheat, not deserving serious attention. They would be wrong. Even in pure science, where exact solutions are highly valued, approximate methods are widely used. Complex X-ray analysis (of proteins, for example) uses computers to compute maps of the molecules, whose shape is fitted by no simple equation. Approximate methods are used for predicting the structure of all atoms or molecules other than the very simplest, as the equations (Schrödinger's equation) cannot be solved exactly by analytic methods for more than two particles. No analytic equations have been found for astronomical problems involving several comparably sized bodies, and approximate numerical solutions rather than exact equations are used to guide space probes (and very exactly too). On the other hand, approximate analytic solutions are used to guide numerical methods. If they were not, purely numerical methods would often defeat the largest computers imaginable.

In applied science, the role of approximate numerical methods is even wider. The airflow over an aircraft, the distribution of heat or sound in a building, the stresses in a proposed dam or bridge, the magnetic field around a new design of motor armature, or the effect of changing the shape of the hull of a ship are all examples where such methods, nowadays using computers, are the only possible approach in practical problems. Engineers have pioneered new kinds of numerical calculation, ahead of the mathematicians. The references in one book* about such methods mention their use for calculations on airflow, vibrations, heat flow, deflection of spars of varying thickness, bridges, electrical networks, stability, torsion in beams or shafts, lubrication, structural frameworks, resonances (of aero engines and aircraft wings), stresses in hooks, shock waves, magnetic fields, flow through pipes and nozzles, and many other problems.

The availability of computers, which quickly do simple repeated calculations of the sort used in the graphical analyses proposed in this course, has made it far more practicable to tackle tough problems by approximate methods. Of course, the exact methods of differential and integral calculus are still valued, though more for their elegance, generality, and compactness than for their precision, for approximate methods can be made as exact as one pleases if one takes enough trouble.

Because computers are so much used for this kind of problem, there is merit in handling some problems in physics teaching in a computational manner rather than an analytic one. Students who meet such methods later on may find them less strange than do others who meet only analytic methods. Indeed, in trials one or two students developed computer programmes to perform numerical integrations. But the main reason for using these methods is that they seem to us to offer a better insight into the meaning of derivatives and the way mathematics models a physical situation than do more formal methods. That is, they are *more* mathematical than is analysis.

Students' book

Question 43 is a problem that is not so easy to solve by analytical methods. It may be useful as an example, but it is not essential.

*Allen, D. N. de G. (1954) *Relaxation methods*. McGraw-Hill.

The smaller the interval Δt, the better the approximation. When solving real problems in this style, engineers and physicists devote much attention to choosing Δt small enough to be just sufficiently accurate, but not so small as to make the job unnecessarily tedious. Pupils are likely to have more confidence in the idea if the problem of sufficiently good approximations is taken seriously.

(Can the apparatus, in fact, detect an error of the size there appears to be between the graph and results found from $s = \frac{1}{2}at^2$?)

How constant acceleration is represented in drawing the graph

In drawing the graph, each new section was drawn at a steeper slope; that is, a larger velocity. Because the acceleration was constant, the slope increased by equal amounts in each step. See figure 29.

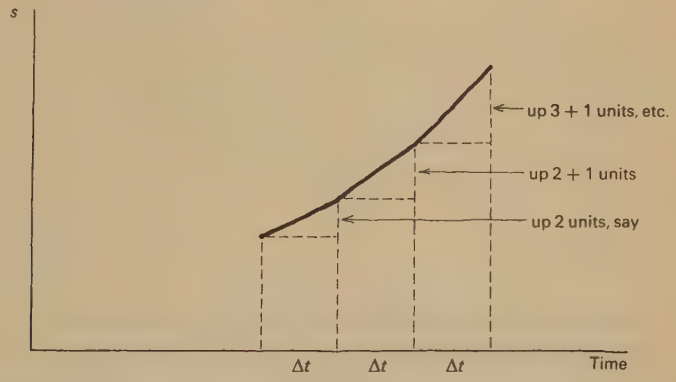


Figure 29

At each moment we worked out the average velocity near that moment, using the recipe $a = 10\ \text{m s}^{-2}$. A line like AB in figure 30 at the right slope was put in, going an extra distance $v\Delta t$. At the next moment, v was larger, say $v + \Delta v$, where Δv is the extra velocity gained in an interval Δt. We drew a line like BC, at the new, larger slope, going a larger extra distance $(v + \Delta v)\Delta t$.

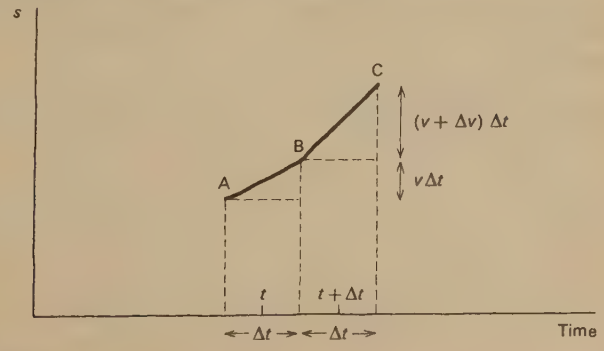


Figure 30

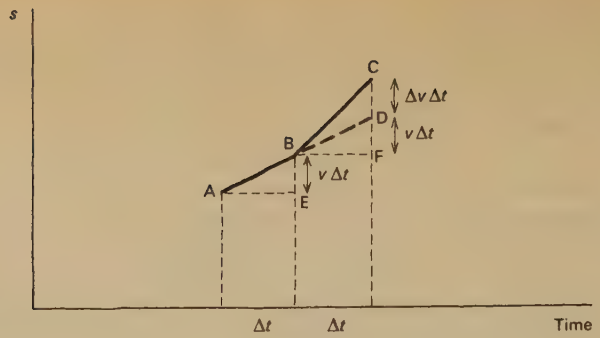


Figure 31

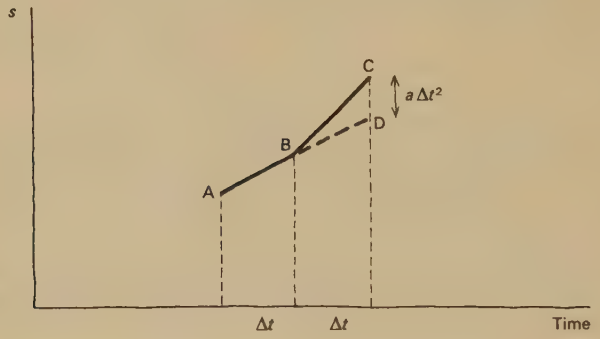


Figure 32

Mathematics and notation

The discussion of graphs and the meanings of $v\Delta t$, $\Delta v\Delta t$, etc., may seem to belong more to mathematics than to physics. This is one of the pieces of mathematics chosen for careful attention in the physics course, because we believe the mathematics of rates of change to be so valuable in pure and applied science that it is worth the trouble.

The Δ notation we regard as useful enough to be seen by all. Some teachers will prefer δ, but as the actual size of the differences is always explicit in the work that follows, one symbol seems to be enough.

Students new to the notation will find it strange that Δs or Δt does not mean Δ *multiplying* s or t, and that Δ cannot be cancelled in an equation like $\Delta s = k\Delta t$. Frequent translation of equations into words will be needed.

It may also be necessary to remark that if an equation relates, say Δs and Δt, the intervals Δs and Δt must be pairs that correspond to one another.

If, as in figure 31, AB is run on at the same slope, to D, then DF = $v\Delta t$ is the extra distance that would have been covered if the velocity had *not* increased. The 'extra extra' bit CD = $\Delta v\Delta t$ is the 'extra extra' distance gone because the velocity *did* increase, by Δv. So CD = $\Delta v\Delta t = a(\Delta t)\,(\Delta t) = a(\Delta t)^2$.

A rule for drawing graphs of acceleration

So there is a simple rule for drawing acceleration graphs. See figure 32. Take the graph AB as found at the last interval, and run on AB straight to D, as if there were no acceleration. Then add an 'extra extra distance' DC, where DC = $a\Delta t^2$. Then BC is the next bit of graph. If the acceleration should vary, the 'extra extra distances' like DC will vary; just work them out from the basic recipe.

Optional : the rule with a Δ notation

For some pupils, the argument in terms of acceleration will be more than enough. Some may like to see how the calculus notation is useful. Δ is used to mean 'small change in . . .'. Over AB, figure 33, the velocity v is the slope of AB, that is $\frac{\Delta s}{\Delta t}$. Then the velocity rises, and the acceleration is

$$\frac{\Delta v}{\Delta t} = \frac{\Delta\left(\frac{\Delta s}{\Delta t}\right)}{\Delta t}$$

Over the time Δt, the distance s changes by more than it would have changed if there had been no acceleration, and Δs in the second interval is larger than Δs in the first by the extra extra distance $\Delta(\Delta s)$. Further, the acceleration is $\frac{\Delta(\Delta s)}{\Delta t^2}$, equal to the rate of change of velocity around B.

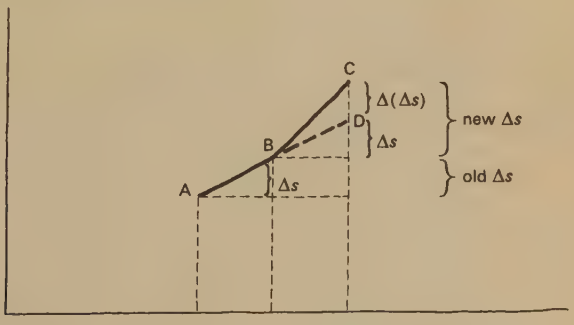


Figure 33

Optional discussion of $\frac{\Delta\left(\frac{\Delta s}{\Delta t}\right)}{\Delta t}$ as an approximation to $\frac{d^2s}{dt^2}$

We think that the handling of graphical solutions is within the capacity of all students, and that many can be led to see the sense of the notation $\frac{d^2s}{dt^2}$ in terms of finite differences. It may be necessary to pass over the crucial matter of whether a limit to $\frac{\Delta^2 s}{\Delta t^2}$ exists as $\Delta t \to 0$. But there are students whose difficulties with notation would take too long to overcome, and they will have to stop a little short of the full course. (The final section will still have plenty to offer them.)

At least the discussion opposite may clarify why one does not write $\frac{ds^2}{dt^2}$ or $\frac{d^2s}{d^2t}$.

Teaching the graphical analysis

Teachers will have to decide the extent to which they will have to exercise their skills of persuasion to carry the class along confidently. Some students will be able to work through the method by themselves, using the *Students' book*, while others will manage best under guidance.

The effort of plotting out much more than a quarter period of the curve is unlikely to be worth while. Errors may accumulate, and as a result the amplitude of the solution may well fluctuate.

Students' book

Question 44 sets out the analysis of $\frac{d^2s}{dt^2} = -10s$ step by step, for use at home or in class. It is the key question in this Part.

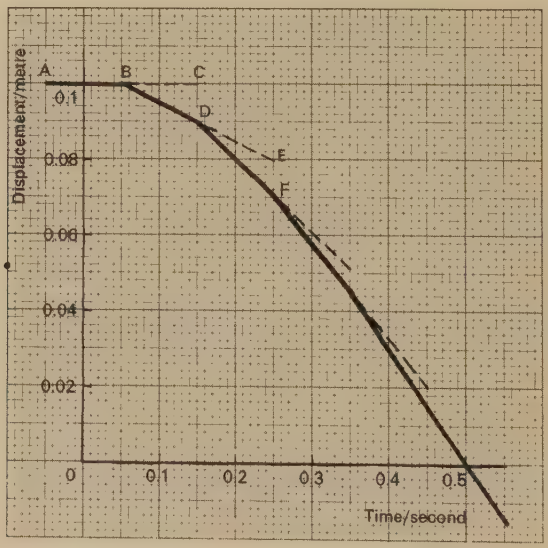


Figure 34
Displacement of a harmonic oscillator, $k/m = 10$.

So we have the same rule: add on an 'extra extra distance' over and above that which would come from taking the graph straight on. The size of the 'extra extra distance' should be:

$$\Delta(\Delta s) \text{ or } \Delta^2 s = (\textit{acceleration})\,\Delta t^2.$$

$\frac{\Delta s}{\Delta t}$ approximates to the velocity v, or $\frac{ds}{dt}$.

$\frac{\Delta^2 s}{\Delta t^2}$ approximates to the acceleration a, or $\frac{d^2 s}{dt^2}$.

Solving the harmonic oscillator equation

The argument can now return from its detour via constant acceleration, equipped with the tools to solve the oscillator equation, with its varying acceleration.

The equation is:

$$a = -\frac{k}{m}s$$

or $$\frac{d^2 s}{dt^2} = -\frac{k}{m}s \qquad \text{with } \frac{k}{m} = 10\ \text{s}^{-2}.$$

It will help the argument if a spring-tethered trolley, as used in experiment 4.12, is on the bench so that it can be moved about and observed as the argument about its motion develops. The argument uses the data for the trolley as recorded in that experiment and these records need to be available. It was arranged that the force needed to pull the trolley aside by one metre and the mass were adjusted so that $k/m = 10\ \text{s}^{-2}$. The trolley should then have oscillated with a period of about 2.0 seconds. We now try to predict and understand that value.

This time the acceleration is not uniform but is given by $a = -10s$. s is the displacement. Again choose time slices of 0.1 second so that the 'extra extra distance' travelled during each will be given by $a(0.1)^2 = -10s(0.1)^2 = -s/10$. Now the displacement graph can be begun, starting from a maximum displacement of, say, $s = 0.1$ metre when $t = 0$ and the velocity is zero.

It is essential to have a simple graph scale, and the best is the simplest, 0.1 metre of graph paper representing 0.1 metre of displacement.

Figure 34 shows the completed graphical solution. The line AB shows the position of the oscillating mass during the interval 0.1 second around $t = 0$. (Of course, it is only exactly right for the instant $t = 0$, that is, mid-way between A and B.) If the velocity remained zero the graph would go on to C at the end of the next 0.1 second slice of time. But the velocity changes during the time BC. Around B the acceleration is 1.0 metre per second each second.

References for teachers

Bork, *Fortran for physics,* is a useful source for schools which have access to a computer and could programme some of this work.
Feynman *et al., The Feynman lectures on physics,* Volume 1, Chapter 9, uses a numerical version of the graphical method, for the harmonic oscillator. Teachers will enjoy it, and some students may.
Sherwin, *Basic concepts of physics,* uses graphical analysis of essentially the same kind as ours.

Alternative arguments for the harmonic oscillator

The two mathematical alternatives are to integrate the equation directly, or to postulate that displacement varies as the cosine of ωt and differentiate twice, obtaining the differential equation back again.

The graphical-numerical method suggested here seems to us to have some advantages over both of these. For both alternatives, the algebraic manipulations are more remote from the physics of the problem than are the numerical steps of the approximate solution, where each step can be closely related to what happens now, to something on the bench. Such arguments may help students to think sensibly about oscillations, and may illustrate directly how the dynamical problem (restoring force proportional to displacement) comes to have a cosine solution.

Similar methods are used in Unit 2, *Electricity, electrons, and energy levels,* and in Unit 5, *Atomic structure,* to develop the exponential solution of a first order differential equation, this and the equation above being the pieces of mathematics which receive extended treatment in the course. The exponential will be used again in the course (Unit 9, *Change and chance*), and the work on oscillations will pay off in the final Unit of the course when it will be possible to obtain, with little trouble, a solution to a simplified version of the Schrödinger equation for a hydrogen atom.

Choice of arguments for a cosine curve and $T = 2\pi/\sqrt{k/m}$

The graphical analysis can predict the shape of the experimental curve, but does not give the analytical form of the solution. As the solution does have a simple form which it is useful to know, other arguments for this are given. Not all students need see all of them.

Argument A: differentiating $\cos \omega t$

This argument produces both the above results, and is brief. Those who can take it may get added value from seeing one of B or C.

Argument B: mapping simple harmonic motion onto a circle

This argument produces the cosine form of the curve, and is simple and practical. Argument D for $T = 2\pi/\sqrt{k/m}$ supplements it.

Argument C: plotting a cosine curve and checking with tables

This argument also produces only the cosine form of the curve, and requires supplementing by argument D. But it can give the right student a sense of mathematical power, by letting him or her generate from first principles a fairly reliable table of cosines as well as a value of π. (Too many students behave as if they think that trigonometric tables are a sort of revealed truth, and that the values of π or of e are mysteries too deep to probe.)

Argument D

This goes again over an idea used earlier, on page 73. It considers the velocities and accelerations, and suggests that if the value of T is halved, the value of k/m must have been quadrupled.

So the 'extra extra distance' travelled in the time interval BC is $-10s(0.1)^2 = -s/10 = 0.01$ metre, since $s = 0.1$ metre at present. CD may be marked off and BD drawn in. In the next time interval, the acceleration is taken as what it is at D, that is $-10(0.09)$ metre per second each second, the distance s now being 0.09 metre. The 'extra extra' step next travelled, EF $= -0.009$ metre, and so on.

Values of s may be read off the graph, rather than calculated at each stage, without serious loss of accuracy. Then at each stage, the next change to be made to the dropping curve is just $-s/10$, where s has been read off from the place last reached. The teacher could take the class as a whole through the plotting, until the graph has cut the axis. It would be well to go on a few steps further, so that the class see that, now s has changed sign, the 'extra extra distances' must keep the graph curving 'inwards'. (The springs pull inwards.)

Carefully drawn, the curve will cut the axis at close to 0.5 second. Any who wish to go on may find it cutting again at 3×0.5, 5×0.5, etc., the periodic time being close to $4 \times 0.5 = 2$ seconds.

How does this compare with the actual period of a tethered trolley for which k/m is arranged to have the numerical value 10? The results of experiment 4.12 should agree with the prediction. The curve should be compared with those drawn from the results of experiment 4.12, one quarter of a cycle being enough. The graphical analysis is capable of predicting both the time of oscillation, and the shape of the displacement–time curve.

A cosine curve

What *is* this curve? Students may know, or be told, that it is a cosine curve, and that its periodic time T is given by $T = 2\pi/\sqrt{k/m}$. An immediate check on the time is possible, for since $k/m = 10\ \mathrm{s}^{-2}$, its square root is nearly equal to π, so that T is close to 2 seconds and the quarter period $T/4$ to 0.5 second.

But there are arguments for both the cosine nature of the curve and the relation between T, k, and m, and students should see some of them.

Argument A : differentiating $\cos \omega t$

Suppose $s = A \cos \omega t$ where A and ω are constants.
Then $\mathrm{d}s/\mathrm{d}t = -\omega A \sin \omega t$
and $\mathrm{d}^2s/\mathrm{d}t^2 = -\omega^2 A \cos \omega t = -\omega^2 s$.
But the problem concerned the equation

$$\mathrm{d}^2s/\mathrm{d}t^2 = -(k/m)s$$

So the equation $s = A \cos \omega t$ is the curve generated, with $\omega^2 = k/m$.

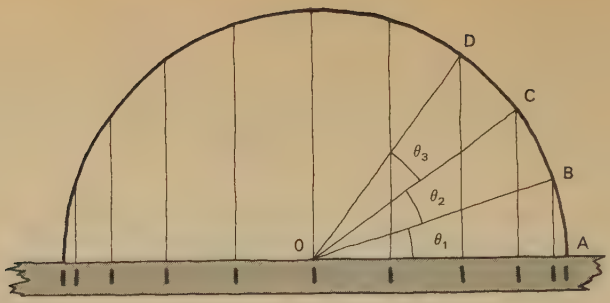
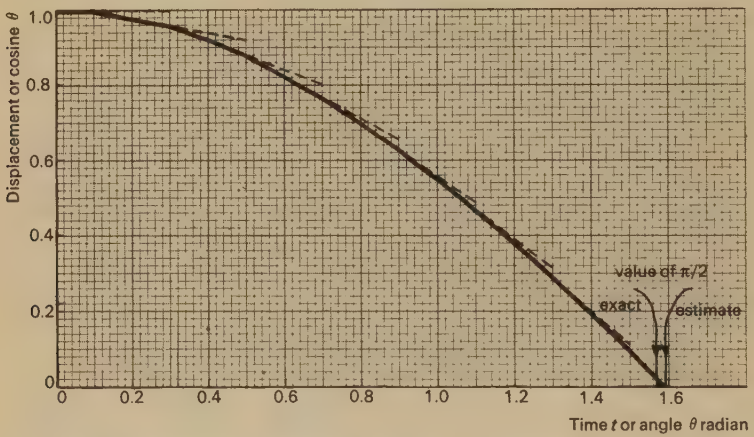


Figure 35
Mapping simple harmonic motion onto a circle.



θ/radian	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6
cos θ (graph $\Delta t = 0.2$)	1.0	0.98	0.92	0.82	0.70	0.55	0.39	0.19	0.0
cos θ (tables)	1.00	0.980	0.920	0.825	0.697	0.540	0.380	0.170	−0.028

Figure 36
Graphical solution of $d^2s/dt^2 = -s$ ($\Delta t = 0.2$).

Students' book

Question 45 goes step by step through the plotting of the cosine curve in figure 36, used in argument C.

Now ωt is the argument of a cosine function and must be an angle, so ω is a rate of change of angle. Since a cosine repeats itself at intervals 2π in angular measure, over a periodic time T, the rate of change of angle $\omega = 2\pi T$.

Thus $k/m = (2\pi/T)^2$

or $T = 2\pi/\sqrt{k/m}$.

Argument B : mapping simple harmonic motion onto a circle

Using a ticker tape or multiflash photograph of the tethered trolley oscillator from 4.12, proceed as indicated in figure 35.

Draw a semicircle 'on' the multiflash photograph, of radius equal to the amplitude of the oscillation. From each 'flash' draw a perpendicular to the diameter to cut the circle at A, B, C, etc. Join these points to O and inspect the angles θ_1, θ_2, etc, which turn out to be very nearly equal to one another.

The displacements from the centre at equal time intervals are in proportion to the cosines of angles which change in equal steps. The time scale of the graph can be 'mapped' onto an angle scale. 'Mapping' in this sense has a definite mathematical meaning. Each value of the time t has (over one cycle) just one corresponding angle, much as each point on the ground has a corresponding point on a map of that country. Some of the corresponding values are:

Angle θ	2π	π	$\pi/2$
Time t	T	$T/2$	$T/4$

$$\theta = \left(\frac{2\pi}{T}\right)t$$

Writing $\left(\frac{2\pi}{T}\right)t$ for θ then gives the harmonic oscillator equation:

$$s = A\cos\left(\frac{2\pi}{T}\right)t.$$

Argument C : plotting a cosine curve and checking with tables

Students can, preferably at home, plot out a second graphical solution. Invite them to try $a = -s$, that is, with $k/m = 1$. See figure 36. Starting with $s = 1$, the values of s correspond very well with values read from a table of cosines against radians. The graph first cuts the axis at or near $t = 1.57$, that is $\pi/2$. They have calculated π too!

Then the same argument as in B relates angle θ to time t, leading again to $s = A\cos\left(\frac{2\pi}{T}\right)t$.

Comparison of argument D with the graph drawing process

Each of the steps **1**, **2**, and **3** opposite has an exact parallel in the graph drawing process. Teachers, and a few students, may like to see how the argument can be developed in terms of graph slopes. Halving the time corresponds to drawing a new graph of the same form as the old one but within a space on the time axis squashed up by a factor two. The scale of the displacement axis may be left unchanged. See figure 37.

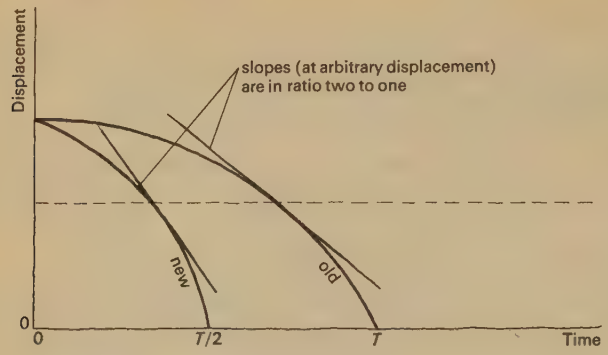


Figure 37
Halving the time of oscillation in graph drawing.

Then the three steps become:

1 The slopes at each displacement must be doubled, for the graph has just been squashed up sideways by a factor two.

2 The slope has to change by twice as much in half the time. The rate of change of slope has quadrupled.

3 The rate of change of slope, represented in the graph drawing process by a series of 'extra-extra distances' (page 83) is proportional to k/m. So k/m must have quadrupled.

Mathematical models

Models of many kinds are mentioned from time to time in the course, for they are often used in physical inquiry. The aim here is to illustrate the relation of a mathematical model to a piece of reality. Some students are prone to suppose that the mathematics is somehow more 'true' or more 'accurate' than the phenomenon, others to think that the model is too idealized to be of real use at all. Both views need some modification.

A mathematical model can be too simple to be useful: a model of the economy in which everyone was imagined to receive the same average wage might well be too severe a distortion, for example. Yet much effort has to go into choosing a model of the right degree of complication. If damping is not too big, the oscillation time of a car axle could be predicted from a model in which damping is not represented. If damping is included, a model with one imaginary source of friction may be adequate, even though the real axle is damped in several ways at once. Also, the less complicated the model, the wider is its likely range of application. But along with any practical use of a model will have to go some thought as to its likely inadequacies.

Students' book

Question 46 gives a discussion of simple harmonic motion as a mathematical model in question and answer form, and could be the basis of class discussion. See also 'Mathematics and physics' in the *Students' book*, which gives two extracts about the use and limitations of mathematical models.

Argument D : halving oscillation time must quadruple k/m

This argument supplements B or C. It has been used before (page 73) in considering the effect of mass and spring constant on oscillation time.

Let k/m be changed so as to halve the oscillation time, keeping to the same amplitude. How must k/m have changed? Consider a quarter of an oscillation.

1 The speed at each moment must have been doubled (same distance, half the time).

2 The double speeds have to be reached in half the time, so accelerations are quadrupled.

3 Acceleration is proportional to k/m, so k/m must have quadrupled when T was halved.

Therefore $T^2 \propto \dfrac{1}{k/m}$

or $T \propto \dfrac{1}{\sqrt{k/m}}$.

But when $k/m = 1$, as in argument C, $T = 2\pi$.

Thus $T = \dfrac{2\pi}{\sqrt{k/m}}$.

If the frequency is required, then as $f = \dfrac{1}{T}$,

$$2\pi f = \sqrt{k/m}.$$

Simple harmonic motion as a mathematical model

Simple harmonic motion, described above, is an idealized kind of motion. Real oscillators – atoms in a crystal or a molecule, car bodies on springs, buildings, and bridges – are likely only to approximate to this motion. Consider whether restoring force is likely to be accurately proportional to displacement, and whether there will be damping to complicate the story, in one or two practical cases. The above arguments illustrate how a mathematical model can be built up, to be used in describing phenomena. Such models are often, as here, strictly limited to ideal cases, and represent real events more or less well. But such seeming inadequacy can be a strength, for the ideal model can be quite simple and can apply to many things. If necessary, more complicated mathematical models can be devised, to cope with damping, for example.

Students' book

Questions 47 to 57 offer a number of examples of uses of $T = 2\pi/\sqrt{k/m}$. We think it important that students should see this result being useful in a wide range of circumstances. Some involve resonance, and could be saved until later in this Unit.

Applications of ideas about oscillations

See Appendix D for brief details about how atomic oscillations are used in spectroscopy to study atomic bonds and identify compounds.

Other applications include the springing of cars, design of loudspeaker cabinets and record player pick-ups, oscillations of car body panels or of aircraft wings, atomic oscillations used as accurate clocks (ammonia, particularly), design of musical instruments, vibration of buildings, and so on. The list can be made very long. The questions mentioned above deal with some of these.

Further details about the order of magnitude calculation of atomic frequency

Spring constant

See *Students' book*, Unit 3, *Field and potential*, page 105. Here, in the section on ionic crystals, a graph of the variation of the net force between ions with spacing is drawn out. Question 42 shows that the force changes at the rate of 42×10^{-9} N for 2.8×10^{-10} m change in spacing; that is at 150 N m^{-1}. Hence the order of magnitude 100 N m^{-1} for the spring constant in the calculation opposite.

Ionic oscillations induced by an oscillating electric field

If the wavelength of radiation falling on the ions is large compared to the spacing (and it turns out to be so), at one instant all the ions in a long row are in a field of the same size and direction. Alternate ions are pushed up or down, depending on the sign of their charge, as shown in figure 38.

A moment later, the field changes direction, having gone through zero, and each ion is pushed or pulled in the opposite direction to that in which it moved previously.

Thus the ions in a row oscillate, with adjacent oppositely charged ions moving out of phase with each other.

Data on absorption by sodium chloride

Slide 4.4 shows a spectrometer absorption curve for a thin layer of solid sodium chloride taken by an instrument capable of recording energy in the infra-red. Unfortunately, the wavelength is too long for it to be possible to repeat the experiment with normal school apparatus.

The usefulness of the oscillation time result ($T = 2\pi/\sqrt{k/m}$; $2\pi f = \sqrt{k/m}$)

It would be disastrous if students supposed that all that has been done is to explain the oscillation time of a trolley between springs, when there are many problems that can be handled using the result obtained. Many are best left for further thought. One example of a use of the result is set out below. It concerns the absorption of light by oscillating ions in sodium chloride, linking with Unit 3 (see 'Ionic crystals').

Order of magnitude calculation of the frequency of atomic oscillations

The average mass m of sodium and chlorine ions is close to 5×10^{-26} kg. The spring constant k of the ionic bond can be obtained from previous work on sodium chloride. It has the order of magnitude 100 N m^{-1}.

Thus k/m is about 20×10^{26} N m^{-1} kg^{-1}. Since $(2\pi f)^2 = k/m$, an estimate for f^2 is about 5×10^{25} s^{-2}, giving an order of magnitude of 10^{13} Hz for the frequency f with which the ions might oscillate. A closer estimate is not worth making, because the frequency will depend on the particular directions in which the ions oscillate, and on which way neighbouring ions are moving.

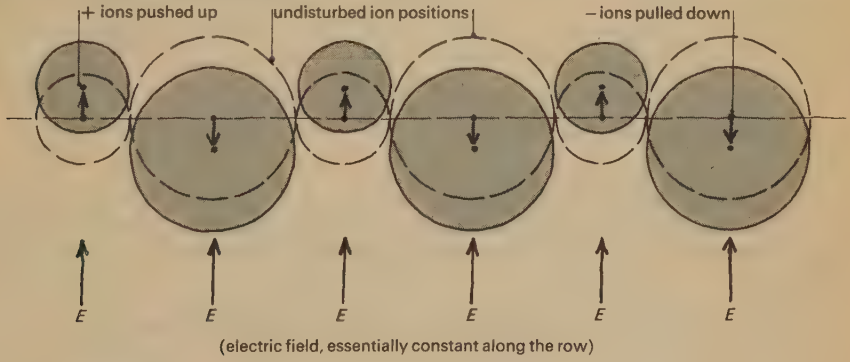


Figure 38
A row of ions in the electric field of long wavelength radiation.

Suppose now that light is, as was suggested in Part One of this Unit, an oscillating electrical disturbance. Then, if light shines on sodium chloride, the ions might be driven into oscillation as suggested in figure 38. If the light has just the right frequency, energy would be absorbed from it by the oscillating ions.

The required frequency is of the order 10^{13} Hz, corresponding to a wavelength of 3×10^{-5} m, which is in the infra-red region of the spectrum. If the experiment is tried, it is found that sodium chloride does indeed absorb infra-red radiation at one particular wavelength. The wavelength is close to 6×10^{-5} m. The order of magnitude of the calculation is right. The discrepancy suggests that the force constant value needs adjusting for the particular kind of oscillation induced by the

Energy of an oscillator

This topic has a brief mention here, partly for completeness, and partly for later use. Later work on reactive circuits will use the idea that the power delivered by an alternating current is proportional to the square of the maximum current, and work on physical optics will involve the idea that the intensity of light on a screen is proportional to the square of the wave amplitude. Finally, in Unit 10, *Waves, particles, and atoms,* an account of wave-particle duality will require the idea that the probability of arrival of photons or electrons at a screen depends on the square of a wave amplitude. The shape of the potential energy graph for a harmonic oscillator may also be used in a brief and simple discussion of the quantum mechanical harmonic oscillator, with its equally spaced energy levels.

For these purposes, the main emphasis should be on energy proportional to $(amplitude)^2$, with a mention of the variation of potential energy as $\frac{1}{2}ks^2$. The kinetic energy as the difference between total energy and potential energy will also be needed for Unit 10.

The discussion also offers an opportunity to revise the conservation of energy, especially as oscillators are among those rare instances where friction can be very small so that kinetic energy + potential energy is very nearly constant.

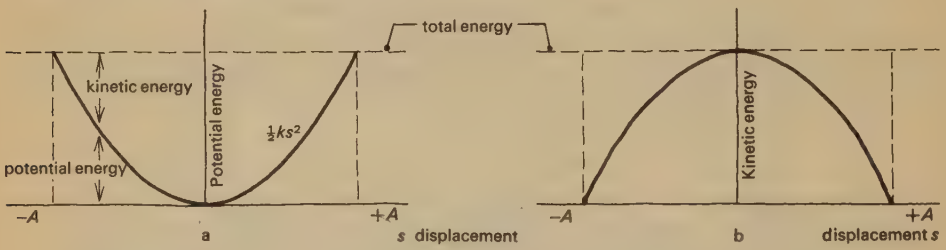


Figure 39

Investigation of resonance

There are not very many occasions in this course when it is suggested that students be left alone with apparatus with the task of thinking for themselves what to do. But there should be a few, for several reasons.

They remind students that doing physics involves thinking for oneself, and that initiative is a valuable thing to develop.

They illustrate the point that one of the hard things in doing physics is to know what to do, not how to do it.

They provide valuable practice for the individual investigations that form a part of the course, and upon which students will be assessed.

They can stimulate students simply to open their eyes and see what happens. (They may well illustrate that what people see depends upon what they expect to see.)

They are a good chance to practise a new vocabulary, using terms like 'frequency', 'amplitude', and 'resonance', when trying to say clearly what one has done and what one has seen.

radiation. (The estimate came from arguments about squeezing the ions together without changing the angles between them; the effect of the radiation is to shift the ions sideways in relation to one another.)

Energy of an oscillator

The oscillating ions in the preceding example remove energy from the infra-red radiation shining on the crystal.

How much energy is stored by an oscillating mass? What happens to the energy during one oscillation? Such questions will bring out the changes from potential energy to kinetic and back again.

The potential energy stored in a spring was discussed in Unit 2. Potential energy $= \frac{1}{2}ks^2$, where k is the spring constant and s the displacement. Figure 39 *a* shows how the potential energy varies with displacement. If the total energy is constant (no damping) the kinetic energy is the difference between total and potential energy. Figure 39 *b* shows how the kinetic energy varies. If A is the amplitude (the maximum displacement), the total energy is equal to $\frac{1}{2}kA^2$. It is useful to list values of potential, kinetic, and total energy at several places in the oscillation cycle, as in figure 40.

displacement	$-A$	0	$+s(<A)$	$+A$
velocity	0	$\pm v$	$+u(<v)$	0
potential energy	$\frac{1}{2}kA^2$	0	$\frac{1}{2}ks^2$	$\frac{1}{2}kA^2$
kinetic energy	0	$\frac{1}{2}mv^2 = \frac{1}{2}kA^2$	$\frac{1}{2}mu^2$	0
total energy	$\frac{1}{2}kA^2$	$\frac{1}{2}kA^2$	$\frac{1}{2}ks^2 + \frac{1}{2}mu^2 = \frac{1}{2}kA^2$	$\frac{1}{2}kA^2$

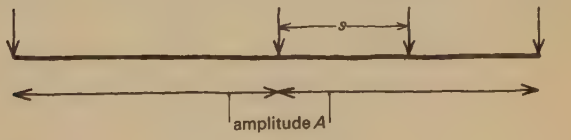


Figure 40
Energies at various stages of an oscillation.

Resonance

One example has already been met: the response of oscillating ions in sodium chloride to an oscillating electric field. Other practical instances of things that can oscillate being driven by an oscillating force abound. In the next experiment, students can investigate some of the many interesting things that happen then.

Teachers will have to find their own tactics to achieve such ends. Setting up one sample apparatus beforehand helps, by eliminating an 'investigation' of how it is meant to be put together or used! It should be clear that measurements are expected, and that they should be made for a previously decided purpose even if the results are surprising and lead off in a new direction. Enough time will be an essential commodity. We suggest two long practical sessions, with a set of apparatus for each pair of students.

Students will still ask, 'But what do you really want us to "discover"?', and it will be necessary to say that it does not matter in the least what they see or fail to see as long as they invent and try some reasonable experiment. This investigation is not about learning some bit of physics, but is about learning to do physics. The following demonstration, 4.14, can look after all the resonant phenomena students must see.

Investigation

4.13 Resonance in a simple system

1024	hacksaw blade oscillator
1053	hacksaw blade (if not with item 1024)
1053	Meccano strips (No. 1 and No. 2a) (if not with item 1024)
507	stopwatch or stopclock
44/1	G-clamp (large)
	The following items should be to hand if required:
501	metre rule
1053	postcard, cork, needle, rubber band
503–6	retort stand base, rod, boss, and clamp 2

One form of the apparatus is illustrated in figure 41, being adapted from a device suggested by Scottish teachers.

The assembly of the oscillator should be clear from the diagram. The blade is driven by the heavy pendulum. Both the mass of the blade and the 1 kg pendulum bob can be adjusted in position to alter their natural frequencies. The degree of coupling may be changed by using different rubber bands (or springs), and damping may be changed by turning the postcard so that it is at right angles to the direction of motion of the blade. The amplitude of the motion of the driver pendulum can be maintained by gently tapping the pendulum strip a little below its support with one finger.

Investigation

4.13 Resonance in a simple system

Students should be given some simple, low frequency resonating system such as the hacksaw blade oscillator illustrated in figure 41, and be asked to think for themselves of observations to make. The central problem is, 'What happens when something (like the blade) that can oscillate on its own at a definite frequency is driven by an oscillating force at a rate which may or may not be the same as the natural blade frequency?'

The term 'resonance' needs to be introduced.

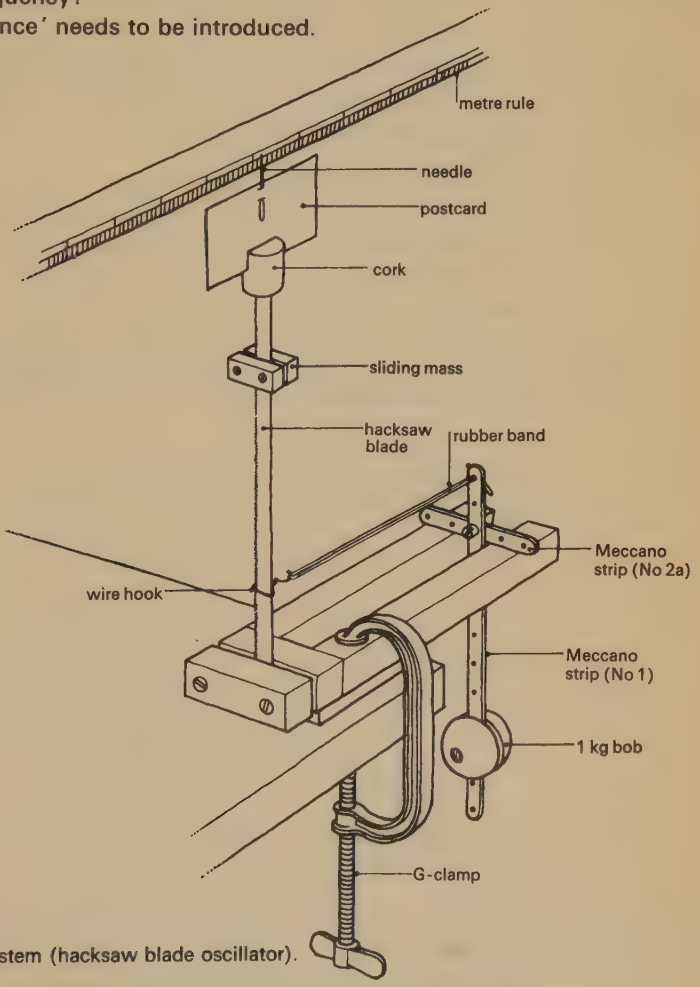


Figure 41
A simple resonant system (hacksaw blade oscillator).

Then it can be explained that this experiment is an exercise in thinking what to do, and doing it. A working physicist spends much of his effort not in deciding how to do some experiment or other, but in wondering what experiment it would be good to do. Some would say that good physicists are those who think of good experiments, rather than those who do experiments well.

Discussion and demonstration

4.14 Barton's pendulums

This demonstration gives the chance to find out what students have observed in experiment 4.13 without them having to make formal reports. Many students will have difficulty in transferring observations from the first situation to this new one, so the practice in doing so is likely to be worth while. It may be pointed out that being able to use ideas on new problems is what understanding amounts to.

133 camera, support, and cable release

171 photographic accessories kit

1054 film, monobath developer, printing paper (P153), paper developer, and fixer

slide projector or other suitable light source

1053 *Materials for constructing the pendulums and support*

AB wooden support, horizontal, 1.5 m long

H, H screw eyes or nails

T thin string or thread

D driver, mass about 0.04 kg

N nylon fishing line

C paper cones; cut 60 mm diameter circles from stiff white paper and then, after cutting along a radius, slide the exposed radial edges round until a cone of double thickness of paper is formed; 'fix' the cone using glue.

plastic curtain rings 20 mm, one per cone

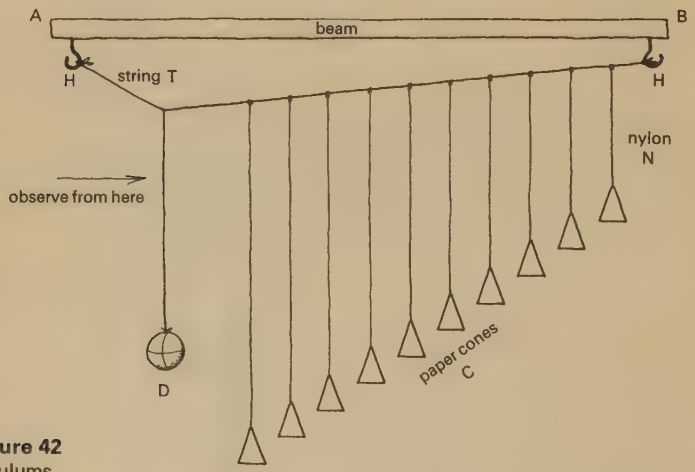


Figure 42
Barton's pendulums.

The construction is shown in figure 42. The wooden support rod should be firmly clamped so as to leave an unobstructed view along the line of pendulums. The lengths of the pendulums can be from about $\frac{1}{4}$ m to $\frac{3}{4}$ m with the driver pendulum $\frac{1}{2}$ m long.

The cone pendulums may be attached to the cross string T by a half-hitch or slip-knot; this makes it easy to adjust the lengths. The pendulums should be as close together as possible. Teachers have also used thread, securing the cones with a blob of Plasticine at the end of the thread.

First it would be sensible simply to observe what happens when the driving force is started up. Then it might be useful to find out how conditions can be varied – what happens if the mass on the blade is shifted? After that, each student will have to decide upon some definite quantitative experiment to do.

Many experiments are possible, and a student ought to be able to hit upon one good thing to try. The natural frequency of the blade can be varied, and the amplitude it builds up to can be plotted against frequency, obtaining a resonance curve. Alternatively, the driver frequency might be varied. The relative movements of driver and driven may interest others. The relative phase of driver and driven changes according to whether the driving frequency is higher than, equal to, or lower than the natural frequency of the driven blade. But these phase differences are not easy to see, and there is no reason why any student should note them at all.

Transient oscillations are rather obvious. After a time the blade oscillates at the driver frequency, with a steady amplitude, but before that its amplitude of oscillation rises and falls. The period of these fluctuations is longer, the smaller the difference between the driving frequency and the natural blade frequency.

Damping effects are also of interest. Heavier damping broadens the resonance curve, and reduces the maximum amplitude reached. It also reduces the time for which transients occur, which is roughly equal to the time for free oscillations of the blade to die away.

No doubt some students will think of sensible experiments that had not been foreseen – varying the strength of the coupling between driver and driven, perhaps. So much the better.

Discussion and demonstration

4.14 Barton's pendulums

When the class and teacher look together at Barton's pendulums, resonance, phase relationships, transients, and damping effects may all be seen again in this fresh situation. The question, 'Did you see anything like this with the hacksaw blade oscillator?' will test how observant students have been.

The large transfer of energy to the resonant pendulum recalls the absorption of radiation of the right frequency by oscillating ions in sodium chloride.

The class should look along the line of pendulums (figure 42) and see how they behave when, with the paper cones at rest, the driver pendulum, D, is released from a widely displaced position. Which pendulum is in resonance with the driver? Are the paper cone pendulums damped? (Yes.) It is difficult to see the general pattern of the cone amplitudes unless a photograph is taken. With the room darkened and the cones illuminated by means of a slide projector, a 'time' exposure may be made of the swinging pendulums. See figure 43.

The demonstration is most effective in a darkened room with only the cones brightly illuminated by the slide projector. Starting with the cones at rest, the driver is released from a widely displaced position. The general pattern of the cone amplitudes is best shown by taking a photograph – a 'time' exposure – with the camera pointing along the line of pendulums from a position on the opposite side to D and at the same level as D. To obtain a sufficient depth of field a small stop (f/8 or f/11) should be used with the camera not too close to the cones. A distance of about 3 m is suitable for a 35 mm camera. An exposure of several seconds will be needed (at least as long as the period of the driver) and the background must be as dark as possible. In interpreting the photographs it should be remembered that there will be some distortion due to the difference in camera distance for the various cones. The effective damping may be reduced by slipping the plastic curtain rings over the cones. This is easily done if the rings are first cut.

To study the phase relationships an instantaneous photograph (1/125 s or less) should be taken just when the driver is at maximum displacement.

Textbooks

For resonance, see:
Feynman *et al.*, *The Feynman lectures on physics* Volume 1; Chapter 23 gives many examples.
Rogers, *Physics for the inquiring mind,* page 190.

For standing waves, see:
PSSC *College physics*, Chapter 8.
PSSC *Physics*, 2nd edition, Chapter 17.
Rogers, *Physics for the inquiring mind,* page 188.

Bishop, *Vibration,* Chapter 3 is first rate on applied problems concerned with resonance.

Appendix D to this *Guide* gives some information about the effects of vibration on man, most of which depend on body resonances.

A standing wave as two travelling waves

The discussion given here assumes students have done the experimental work in Part One on interference between two waves travelling in opposite directions, and is therefore quite brief. It may need to be extended if that work was curtailed, or if the class lacks the rather wide experience of wave behaviour from Nuffield O-level Physics.

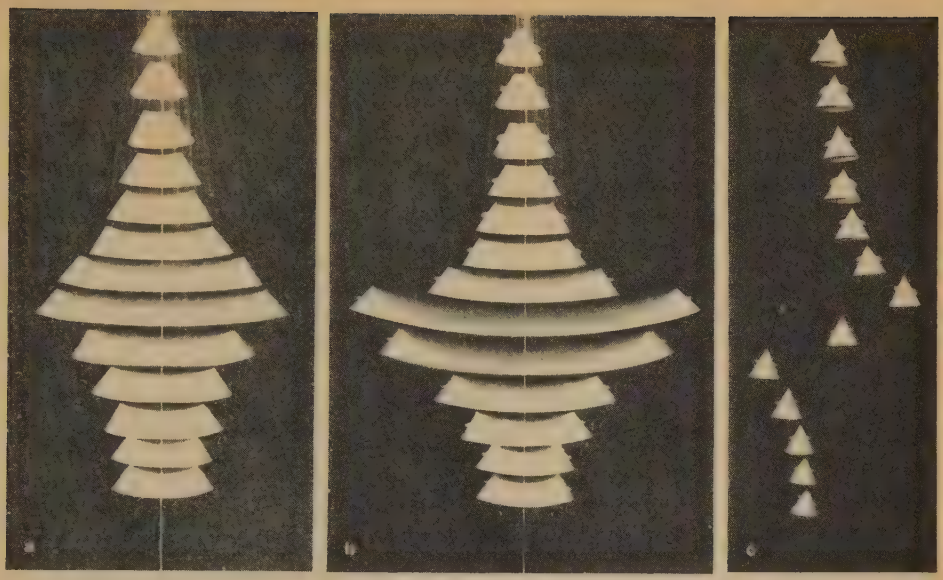


Figure 43
Photographs of Barton's pendulums.
a Time exposure (damped).
b Time exposure (less damped).
c Instantaneous.

To reduce the effect of damping, 20 mm plastic curtain rings may, after being cut, be slipped over the cones. Another photograph is now taken. An instantaneous photograph, taken just when the driver reaches its maximum displacement, reveals the phase relations between driver and driven pendulums, also shown in figure 43.

This demonstration is exceptionally good for illustrating the phase relationships. The relationships do not matter very much yet, but the term 'phase' is vital, and the demonstration brings out its meaning very well.

Standing waves

A concluding discussion of standing waves, which may be looked at from the point of view of oscillations, resonance, and interference, offers a chance to draw together the threads in this Unit. Many oscillations of practical importance are modes of standing wave systems (rather than simple masses and springs), so references to applications can again be made here.

A series of demonstrations is suggested. The more dramatic and polished they are, the better. A darkened room with carefully directed light will make a big difference to the impact.

Demonstration

4.15 Standing waves on springs and strings

4.15a Waves along a spring

101 large Slinky
or
1013 long spring

44/2 G-clamp (small)

One end of the spring needs to be tied tightly to a firm support such as a G-clamp on a rigid bench. The spring should be tensioned and a short train of transverse waves sent along it by rapidly shaking the free end. Standing waves appear when the reflected front of this wave train is superimposed on the tail which has not yet been reflected.

4.15b The resonance of a particular length of spring

The apparatus is as for 4.15a. See also Nuffield O-level Physics *Guide to experiments V*, experiment 94.

Both ends of the spring should be fixed as described above or held firmly with the spring in tension. The spring should be oscillated by shaking it a little near one of the ends, that is, near a node. As the oscillation frequency is increased, large standing waves appear at certain frequencies, the number of loops increasing as the frequency is raised. Shaking it at other frequencies should be tried.

4.15c Standing waves on a rubber cord

1009 signal generator

1060 vibrator

134/2 xenon flasher

1055 rubber cord ($\frac{1}{2}$ m long, 3 mm square cross-section)

503–6 retort stand base, rod, boss, and clamp *2*

121 metal strips (as jaws) *2 pairs*

44/1 G-clamps (large) *2*

1000 leads

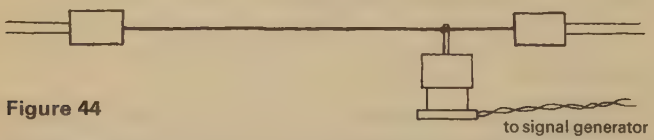


Figure 44

The ends of the rubber cord are held by the metal strips in the retort stand clamps, the retort stands being clamped to the bench so that the rubber cord is stretched to about 1 m length. The vibrator is linked to the cord, a few cm from one end, by a short length of wire (22 s.w.g.) twisted round the cord and fastened to the vibrator. With the signal generator on full sine wave output (low impedance), the frequency should be slowly increased from 10 Hz to 100 Hz, in which range there should be 4 or 5 resonant frequencies. It helps to have white bands painted on the cord at regular intervals along it, and to observe the motion under stroboscopic illumination.

The demonstration can also be done with string (see Nuffield O-level Physics *Guide to experiments V*, experiment 97). One end of the string is tied to the vibrator. The other end, after passing over a pulley, carries a load of 200 g. The vibrating length should be 1 m.

Demonstration

4.15 Standing waves on springs and strings

4.15a Waves along a spring

The purpose is to draw attention to what should have emerged from experiment 4.5a. When a dozen or so rapid oscillations are sent down a long spring, a standing wave appears briefly where the reflected first few waves travel back through the last few waves which have not yet reached the end.

The standing wave is so called because it does not look as if it is travelling in either direction along the spring. Yet it is the result of two similar waves travelling along. But they travel in opposite directions. Figure 46 (page 104) shows how a standing wave develops as two such waves come to overlap each other. For teaching, a better device than such a diagram is the pair of 'plastic waves' (item 126) laid on top of one another and slid along millimetre by millimetre, or a pair of wave strips used on an overhead projector. With both waves moving, there are fixed places where the waves superpose to give zero effect at all times. There are other fixed places where the waves superpose to give an oscillation having twice the amplitude of either wave alone.

It may also help to recall, or show again, standing waves along the line joining a pair of dippers in a ripple tank.

4.15b The resonance of a particular length of spring

When the spring rests on the bench, waves are quickly damped out. But if all of it is supported between two points well above the bench, waves produced by shaking the spring are reflected repeatedly at the ends. For certain frequencies the shaking exactly coincides with waves already going to and fro along the spring, so that the standing waves continue building up to a big amplitude. This resonance of standing waves in constricted spaces is what makes standing waves important. What conditions govern the frequencies at which it happens? A more easily controllable arrangement is needed.

4.15c Standing waves on a rubber cord

As the signal generator's frequency is raised, the cord resonates first in one, then in two, then in three sections, and so on. What is the relationship between the frequencies at which these modes of vibration are most pronounced?

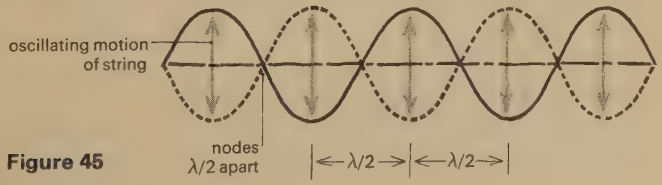


Figure 45

(See also figure 46.)

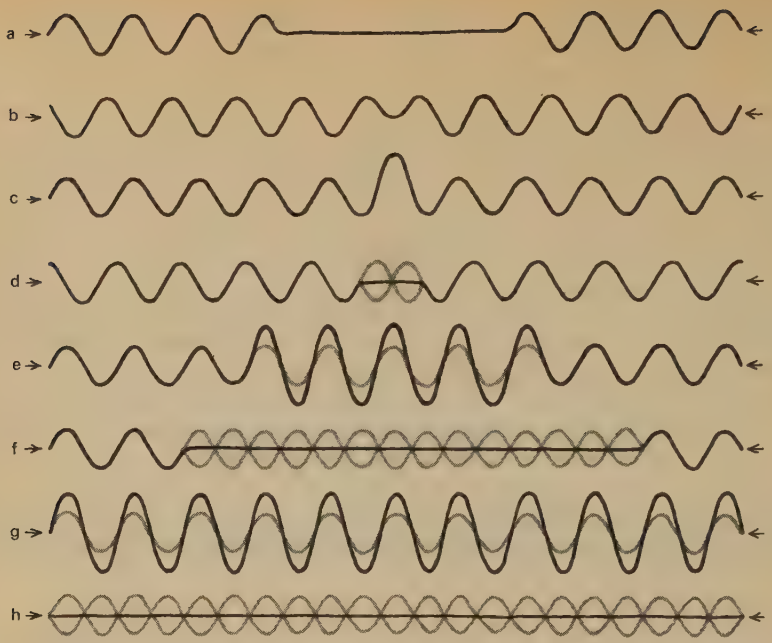


Figure 46
Superposition of two travelling waves.

Trains of waves with the same amplitude and frequency approach each other from opposite sides of the diagram in figure 46.

The full wave profile shows the resultant; the fainter profiles the position of each wave where the two trains overlap.

Within the overlap region there is a standing wave.

The wave produced by the vibrator in figure 44 must take a certain period of time to travel to one end, then to the other end, and back to it again. If this period coincides with the period of the vibrator, we may expect resonance, because the vibrator is just sending off a second wave when the first is about to go on its next trip. But if the vibrator sends off exactly two or exactly three (or exactly any number) of waves in each period of the cord, each wave will be reinforced when it passes again travelling in the same direction. So we may expect a fundamental resonant frequency f and other resonant frequencies $2f$, $3f$, . . . , nf (called harmonics). Readings on the scale of the signal generator should support this. And for, say, the fifth harmonic $5f$, the cord vibrates in five sections (figure 45) separated by four motionless points (nodes).

Resonance

Standing waves are a particular form of interference, seen where two similar trains of waves pass through one another going in opposite directions. Resonance, in the sense of a slow building up of a big localized store of energy, does not occur if the waves can travel on indefinitely. But if the waves cannot get out of a limited space, and the two trains are simply reflections of the same original waves, the energy is imprisoned with the waves and all the normal effects of resonance are seen. There can be standing wave resonance not only in springs but in many other situations, sometimes in two or three dimensions, with boundaries which involve or do not involve phase changes on reflection. Students should not be advised to remember details, but a few examples will come later in the course.

Many waves mutually cancelling

The discussion above suggests why the resonating string responds, but not so clearly why, off resonance, there is practically zero amplitude, despite the motion of the driving force. This point need not be pursued, unless students raise it.

Suppose that the string is one metre long, that it is driven at 100 Hz, but that in 1/100 s waves on it travel down the string and back to within one centimetre of the end. Let the damping be small so that a wave persists for at least a second. Then to the wave generated by the driver at any instant there must be added at least 100 previous waves, each out of step with the one before it by one centimetre.

At the moment when the driver reaches a maximum, the maximum of the wave emitted 100 cycles earlier will be at the far end of the string, having shifted 100 one-centimetre steps. Its minimum will be at the driver, since the wavelength is nearly two metres. Thus this wave will cancel the new wave from the driver.

Similarly every wave persisting on the string is cancelled by one emitted earlier. It further follows that the sharpness of resonance depends directly on the lack of damping. If the waves die out in the time for N cycles, the string will respond at any length differing from a resonant length L by less than about L/N.

Demonstration

4.16 More complicated standing waves

A selection from the following should be made. Only the major pieces of apparatus required are detailed.

4.16a The Kundt dust tube

Any of the usual arrangements may be used, but the following is convenient:

1009 signal generator

1051 small loudspeaker (about 60 mm diameter)

1055 measuring cylinder (100 cm^3)

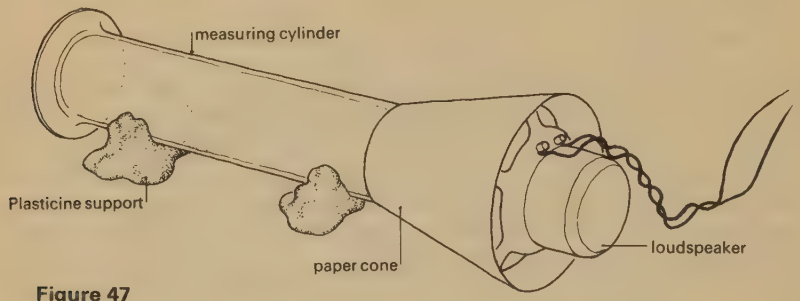


Figure 47

The measuring cylinder is dried and cork dust, made by filing a cork, introduced into it. The cylinder is arranged horizontally and tapped so that a *thin* layer of cork dust forms along the bottom. When the signal generator, set to give an output of about 1 W, is tuned through the 1 kHz to 10 kHz range the cork dust will show the positions of nodes and antinodes.

4.16b Longitudinal standing waves in rods

A rod, about 10 mm diameter, and about 1.5 m long, of glass, steel, or brass, is clamped at its midpoint and stroked with a wet or rosined cloth. Suitable clamping devices are probably available but if not a G-clamp may be used as shown in figure 48.

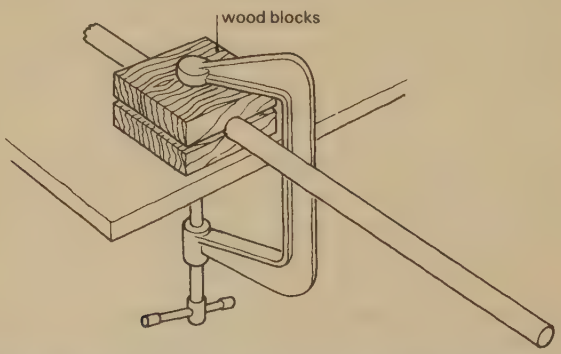


Figure 48

The speed of longitudinal waves in the rod may be found but students should be left to find the frequency without prompting. Comparison with the note from a signal generator is a suitable method.

Demonstration

4.16 More complicated standing waves

A selection from the following suggestions can be used to illustrate that standing waves are to be found in many situations. The waves are in general more complex than those on a string, but certain features remain the same. These, which can be pointed out as the demonstrations are shown, are:

1 The patterns depend on the frequency, there being more nodes or nodal lines for high frequencies (short wavelength).

2 There are a series of definite modes of oscillation, at each of which the response is large (resonance).

3 The standing waves have to 'fit' into the system, whether it has one or more dimensions. Reflected waves from the boundaries interfere with waves travelling towards the boundaries. As a general rule, wave-carrying systems with edges exhibit standing waves. Physicists now think of electrons held inside atoms as being like waves held inside the atom.

As above, polished and effective demonstration will contribute much to the impact of these experiments.

4.16a The Kundt dust tube

A Kundt dust tube excited by a small loudspeaker or vibrator shows a series of resonant frequencies, also in arithmetic sequence. (An interested student could calculate whether the velocity of sound in a tube differs appreciably from that in open air.)

4.16b Longitudinal standing waves in rods

A steel, brass, or glass rod may be clamped at the middle and excited by stroking with a wet or rosined cloth. (The velocity of sound in the rod may be calculated if a student can solve the problem of measuring the frequency.)

4.16c **Vibrations of circular wire rings**

1009 signal generator

1060 vibrator

134/2 xenon flasher

1054 copper wire (20 s.w.g.)

A length of wire (about 1 m) is formed into a circle and attached to a vibrator so that it stands vertically. If the two ends of the wire are made into loops they may be anchored between washers on the vibrator shaft. Copper wire, 20 s.w.g., is satisfactory but the large amplitude standing waves at lower frequencies tend to deform the ring. Thinner steel wire would be better. 10 mm wide strip of 10.05 mm thick steel works very well with a circle about 0.1 diameter. (See *School Science Review,* **47**, No. 162, March 1966, pages 539–543.)

4.16d **Longitudinal standing waves**

1009 signal generator

1060 vibrator

134/2 xenon flasher

1013 long spring

501 metre rule

With the vibrator on its side, one end of the spring is attached to the vibrating element by means of string or a wire loop. A length of about 0.3 m of spring should be stretched to about 0.50 m (these distances are not critical). The hand holding the spring may be rested on a metre rule which at the other end acts as a stop to prevent the vibrator sliding along the bench. The signal generator low impedance output terminals should be used, at full output, with the frequency being increased from about 20 Hz to several hundred hertz. The standing waves should be viewed stroboscopically. The spring shows standing waves at frequencies in arithmetic progression.

4.16e **Vibrations in a rubber sheet**

1009 signal generator

1044 large loudspeaker

134/2 xenon flasher

1053 sheet of rubber

503 retort stand base *2*

1076 big metal ring

A sheet of rubber is stretched over the ring. This is placed over the loudspeaker driven by the signal generator. Lines drawn with a ball point pen on the rubber sheet show the changes in vibration pattern as the oscillator frequency is altered. It is worth while to view the diaphragm under strobe illumination. Details of a demonstration of these standing waves will be found in the *School Science Review,* **50**, No. 173, June 1969, page 930.

See also Unit 10, experiment 10.8.

4.16c Vibrations of circular wire rings

Circular rings of various diameters may be vibrated along a diameter using a vibrator and oscillator. The frequencies of normal modes can be found. A stroboscope may be used with effect.

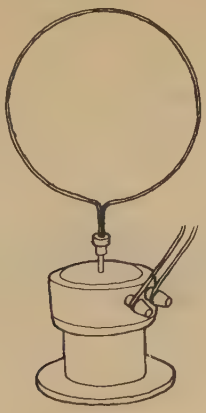


Figure 49

4.16d Longitudinal standing waves

A vibrator drives a long spring into longitudinal standing waves.

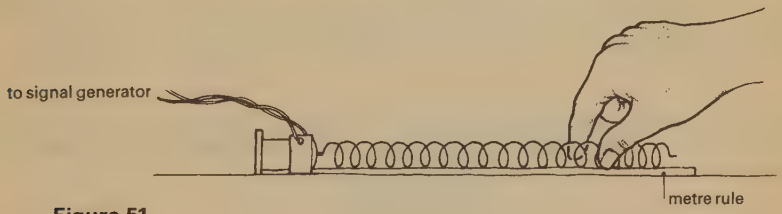


Figure 51

4.16e Vibrations in a rubber sheet

A rubber diaphragm stretched over a solid ring can be excited by placing it over a loudspeaker. Lines drawn on the rubber will show the vibration patterns. Additionally, a stroboscope can be used to see the detailed motion. Again, a series of normal mode frequencies appears.

4.16f **Chladni figures**

1009 signal generator

1060 vibrator

Standard apparatus may be available but, if not, square or round plates may be cut from thin metal and attached centrally to the vibrator. Fine sand may be used to show the vibration patterns as the frequency of the oscillator is altered.

4.16g **Vibrations of a loudspeaker cone**

1009 signal generator

1044 large loudspeaker (exposed cone)

134/2 xenon flasher

The loudspeaker is connected to the signal generator so that the cone vibrates in a vertical direction. A few grains of semolina placed in the cone may show the resonances up clearly, as the generator frequency is changed. The cone should also be viewed under stroboscopic illumination as the frequency alters.

4.16h **Standing waves in a round bowl**

1009 signal generator

1060 vibrator

149 Petri dish from electric fields apparatus

Ring patterns can be produced by using a vibrator with a dipper attached to it. It should be used at a low frequency to generate waves in a Petri dish containing a little water. The dish may be supported with a screen about 0.1 m below it and with a 12 V lamp above it, as a miniature ripple tank.

Standing waves can be set up in a larger bowl (such as a plastic washing-up bowl) by using the wooden block supplied with item 100/2. See also Nuffield O-level Physics *Guide to experiments V*, experiment 95.

4.16i **Standing waves in a rectangular tank**

100/2 large rectangular transparent tank

See Nuffield O-level Physics *Guide to experiments V*, experiments 93c and 94c. The generation of the 'slopping' mode should also be shown by tilting the tank momentarily.

4.16f Chladni figures

Thin metal plates, square or round, can be driven at the centre by a vibrator, and sand can be used to observe the vibration patterns or Chladni figures. The resonant oscillations of car door or body panels are of this general kind.

4.16g Vibrations of a loudspeaker cone

A loudspeaker whose cone may be watched under stroboscopic illumination is driven by an oscillator, and resonance observed.

4.16h Standing waves in a round bowl

Water in a round plastic bowl may be excited into several kinds of standing wave motion. A vibrator can be used to produce ring patterns of waves. The water may also oscillate with several kinds of sideways 'slopping' motions, or waves may be sent round the perimeter of the bowl.

4.16i Standing waves in a rectangular tank

(Perhaps for home experimenting.) Water in a rectangular pan can be excited into slopping modes (figure 50). (This mode is easy to start off, which is why it is hard to carry pans of water.)

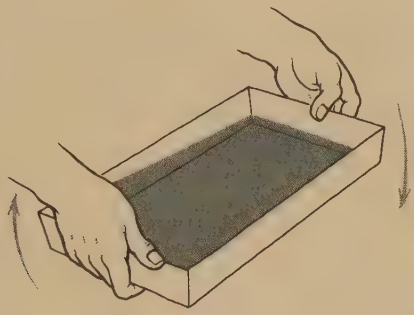


Figure 50

Such standing waves (called seiches) sometimes appear in flat-bottomed lakes.

4.16j **Standing waves – musical instruments**

64 oscilloscope

157 microphone

1035 pre-amplifier (if necessary)

assorted musical instruments (provided by students)

The wave-form of the musical notes produced by various instruments should be exhibited on the CRO. It is best to tape record the instruments beforehand, though the recorder may modify the characteristics of the sounds.

Film loops

As optional extras try:
'Vibrations of a drum.'
'Soap film oscillations.'
'Wind-induced oscillations.'

Reading

Hutchins, 'The physics of violins' (reprint).

4.16j Standing waves – musical instruments

Musical instruments may be shown. A flute is almost a simple pipe, while other wind instruments are more complex. The effective length is changed by opening holes (stops). Stringed instruments have standing waves on the strings, but the wooden structure and air inside also resonate, giving the instrument its particular quality by reinforcing many but definite frequencies.

The usefulness of the standing wave idea

Engineers continually deal with standing waves of one sort or another. Anything that can vibrate and has edges may have a standing wave on it. So the shaft driving a ship's propellers, or turning a turbine, can go into a standing wave oscillation, flexing as it turns.

The wings of an aircraft will also flex like a springy ruler. Two-dimensional standing waves are likely wherever flat panels can vibrate, so they matter to the motor and to the building engineer. Three-dimensional standing waves are a problem for acoustic engineers. A good example is a loudspeaker cabinet enclosing a volume of vibrating air.

Physicists, too, continually deal with standing waves. In this course, in Unit 10, the idea will be used to explain why it is that atoms have definite energy levels, by thinking of electrons trapped in an atom as like a standing wave. But standing waves appear in many other parts of physics. For example, radio waves can form standing waves inside metal cavities, and they have been used both to make very accurate measurements of the velocity of the waves, and in the design of powerful high frequency generators of microwaves.

Appendices

Appendix A
Frequencies of transmissions

The following information may be of use to teachers whose students attempt experiment 4.1c.

Radio stations transmitting on v.h.f.

The powers given are an approximate guide to the radio energy emitted from the aerial, for each programme transmitted. BBC local radio stations are identified by the name of the area or town they serve.

Stations	Frequencies/MHz Radio 2	Radio 3	Radio 4	local	Local power
London and South east					
Oxford	89.5	91.7	93.9		22 kW
Swingate	90.0	92.4	94.4		7 kW
Wrotham	89.1	91.3	93.5		120 kW
BBC Radio London				95.3	16.5 kW
BBC Radio Medway				97.0	5.5 kW
BBC Radio Oxford				95.0	4.5 kW
Midlands					
Sutton Coldfield	88.3	90.5	92.7		120 kW
Churchdown Hill	89.0	91.2	93.4		24 W
Hereford	89.7	91.9	94.1		25 W
Northampton	88.9	91.1	93.3		60 W
BBC Radio Birmingham				95.6	5.5 kW
BBC Radio Derby				96.5	5.5 kW
BBC Radio Leicester				95.2	140 W
BBC Radio Nottingham				94.8	140 W
BBC Radio Stoke-on-Trent				94.6	2.5 kW
East Anglia					
Peterborough	90.1	92.3	94.5		20 kW
Cambridge	88.9	91.1	93.3		20 W
Tacolneston	89.7	91.9	94.1		120 kW
South					
Rowridge	88.5	90.7	92.9		60 kW
Brighton	90.1	92.3	94.5		150 W
Ventnor	89.4	91.6	93.8		20 W
BBC Radio Brighton				88.1	75 W
BBC Radio Solent				96.1	5 kW

Stations	Frequencies/MHz Radio 2	Radio 3	Radio 4	local	Local power
West					
Wenvoe	89.95	96.8	92.125		120 kW
Barnstaple	88.5	90.7	92.9		150 W
Bath	88.8	91.0	93.2		35 W
Oxford	89.5	91.7	93.9		22 kW
BBC Radio Bristol				95.4	5 kW
South west					
Les Platons	91.1	94.75	97.1		1.5 kW
North Hessary Tor	88.1	90.3	92.5		60 kW
Okehampton	88.7	90.9	93.1		15 W
Redruth	89.7	91.9	94.1		9 kW
Isles of Scilly	88.8	91.0	93.2		20 W
North					
Belmont	88.8	90.9	93.1		8 kW
Holme Moss	89.3	91.5	93.7		120 kW
Scarborough	89.9	92.1	94.3		25 W
Sheffield	89.9	92.1	94.3		60 W
Wensleydale	88.3	90.5	92.7		25 W
BBC Radio Humberside				95.3	4.5 kW
BBC Radio Leeds				94.6	140 W
BBC Radio Sheffield				88.6	30 W
North west					
Holme Moss	89.3	91.5	93.7		120 kW
Douglas	88.4	90.6	92.8		6 kW
Kendal	88.7	90.9	93.1		25 W
Morecambe Bay	90.0	92.2	94.4		4 kW
Windermere	88.6	90.8	93.0		20 W
BBC Radio Blackburn				96.4	1.5 kW
BBC Radio Manchester				95.1	4 kW
BBC Radio Merseyside				95.85	2.5 kW
North east					
Pontop Pike	88.5	90.7	92.9		60 kW
Swaledale	89.6	91.8	94.0		35 W
Weardale	89.7	91.9	94.1		100 W
Whitby	89.6	91.8	94.0		40 W
Sandale	88.1	90.3	94.7		120 kW
BBC Radio Durham				94.5	2.6 kW
BBC Radio Newcastle				95.4	3.5 kW
BBC Radio Teesside				96.6	5 kW

Stations	Frequencies/MHz Radio 2	Radio 3	Radio 4	local	Local power
Scotland					
Kirk o'Shotts	89.9	92.1	94.3		120 kW
Ashkirk	89.1	91.3	93.5		18 kW
Campbeltown	88.2	90.4	92.6		35 W
Forfar	88.3	90.5	92.7		10 kW
Lochgilphead	88.3	90.5	92.7		10 W
Perth	89.3	91.5	93.7		15 W
Pitlochry	89.2	91.4	93.6		200 W
Toward	88.5	90.7	92.9		250 W
Meldrum	88.7	90.9	93.1		60 kW
Bressay	88.3	90.5	92.7		10 kW
Grantown	89.8	92.0	94.2		350 W
Kingussie	89.1	91.3	93.5		35 W
Orkney	89.3	91.5	93.7		20 kW
Thrumster	90.1	92.3	94.5		10 kW
Rosemarkie	89.6	91.8	94.0		12 kW
Ballachulish	88.1	90.3	92.5		15 W
Fort William	89.3	91.5	93.7		1.5 kW
Kinlochleven	89.7	91.9	94.1		2 W
Melvaig	89.1	91.3	93.5		22 kW
Oban	88.9	91.1	93.3		1.5 kW
Penifiler	89.5	91.7	93.9		6 W
Skriaig	88.5	90.7	92.9		10 kW
Sandale	88.1	90.3	92.5		120 kW
Wales					
Blaenplwyf	88.7	90.9	93.1		60 kW
Dolgellau	90.1	92.3	94.5		15 W
Ffestiniog	88.1	90.3	92.5		50 W
Machynlleth	89.4	91.6	93.8		60 W
Haverfordwest	89.3	91.5	93.7		10 kW
Llanddona	89.6	91.8	94.0		12 kW
Betws-y-Coed	88.2	90.4	92.6		10 W
Llangollen	88.85	91.05	93.25		10 kW
Wenvoe	89.95	96.8	94.3		120 kW
Brecon	88.9	91.1	93.3		10 W
Carmarthen	88.5	90.7	92.9		10 W
Llandrindod Wells	89.1	91.3	93.5		1.5 kW
Llanidloes	88.1	90.3	92.5		5 W
Northern Ireland					
Divis	90.1	92.3	94.5		60 kW
Ballycastle	89.0	91.2	93.4		40 W
Brougher Mountain	88.9	91.1	93.3		2.5 kW
Kilkeel	88.8	91.0	93.2		25 W
Larne	89.1	91.3	93.5		15 W
Londonderry	88.3	90.55	92.7		13 kW
Maddybenny More	88.7	90.9	93.1		30 W
Newry	88.6	90.8	93.0		30 W

Television channels and nominal carrier frequencies
(U.K. allocations)

Band I

Channel	Carrier frequencies/MHz	
	Vision	Sound
1	45.0	41.5
2	51.75	48.25
3	56.75	53.25
4	61.75	58.25
5	66.75	63.25

Band III

Channel	Carrier frequencies/MHz	
	Vision	Sound
6	179.75	176.25
7	184.75	181.25
8	189.75	186.25
9	194.75	191.25
10	199.75	196.25
11	204.75	201.25
12	209.75	206.25
13	214.75	211.25

Band IV

Channel	Carrier frequencies/MHz	
	Vision	Sound
21	471.25	477.25
22	479.25	485.25
23	487.25	493.25
24	495.25	501.25
25	503.25	509.25
26	511.25	517.25
27	519.25	525.25
28	527.25	533.25
29	535.25	541.25
30	543.25	549.25
31	551.25	557.25
32	559.25	565.25
33	567.25	573.25
34	575.25	581.25

Band V

Channel	Carrier frequencies/MHz	
	Vision	Sound
39	615.25	621.25
40	623.25	629.25
41	631.25	637.25
42	639.25	645.25
43	647.25	653.25
44	655.25	661.25
45	663.25	669.25
46	671.25	677.25
47	679.25	685.25
48	687.25	693.25
49	695.25	701.25
50	703.25	709.25
51	711.25	717.25
52	719.25	725.25
53	727.25	733.25
54	735.25	741.25
55	743.25	749.25
56	751.25	757.25
57	759.25	765.25
58	767.25	773.25
59	775.25	781.25
60	783.25	789.25
61	791.25	797.25
62	799.25	805.25
63	807.25	813.25
64	815.25	821.25
65	823.25	829.25
66	831.25	837.25
67	839.25	845.25
68	847.25	853.25

Appendix B
Measuring the speed of microwaves

The measurement is of the time taken for a very short pulse of microwaves to travel the known distance to a mirror and back again to the receiver. If the total distance travelled is 60 metres, the time taken is 0.2 μs, which is only just measurable on an oscilloscope whose fastest calibrated timebase is 1 cm μs^{-1}. The practical problem is to get a sufficient strength of microwaves back to the receiver, amplify it, and apply it to the oscilloscope, without any signal getting into the receiving end but that which has travelled the full distance.

The transmitter

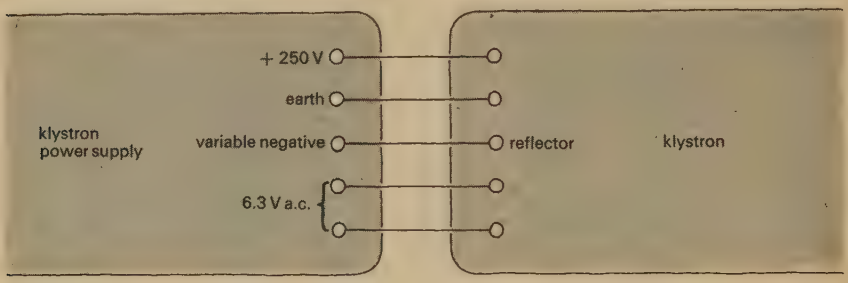


Figure 52

A klystron is supplied with alternating heater current and steady current of the order of milliamps at a high voltage (figure 52). The output is controlled by the degree of negative potential applied to an electrode called the reflector, relative to the klystron's other electrodes, the current taken by the reflector being very small. For most values of the reflector potential the klystron emits no microwaves (figure 53). The difference between no emission and full emission corresponds to quite small changes in the reflector voltage.

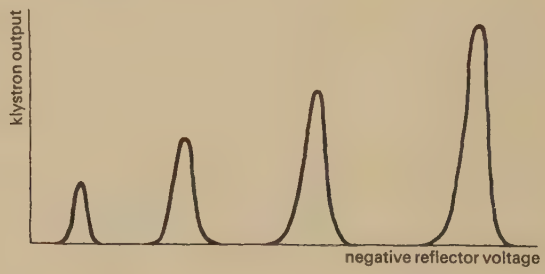


Figure 53

The klystron and its power supply can probably be modulated at some audio frequency or be used to produce microwaves of constant amplitude. The modulation may be produced by an oscillator circuit (inside the power supply) which can sweep the reflector voltage across 50 volts or so. The steady

microwaves are produced with a steady reflector voltage, adjusted to a suitable value by means of a potentiometer in the power supply.

Pulsing the transmitter

The 200 kHz pulse generator (item 1031) contains a multivibrator, rectifier, and *RC* circuit which will produce very brief pulses across a resistor. Figure 54 shows a possible circuit in which the pulses are produced across a 2.7 kΩ resistor. If this resistor is put into the connection between the klystron's reflector and its point on the power supply (which is set to produce a steady reflector voltage and adjusted until the voltage is a suitable one), the klystron will emit brief pulses which can be used for the experiment.

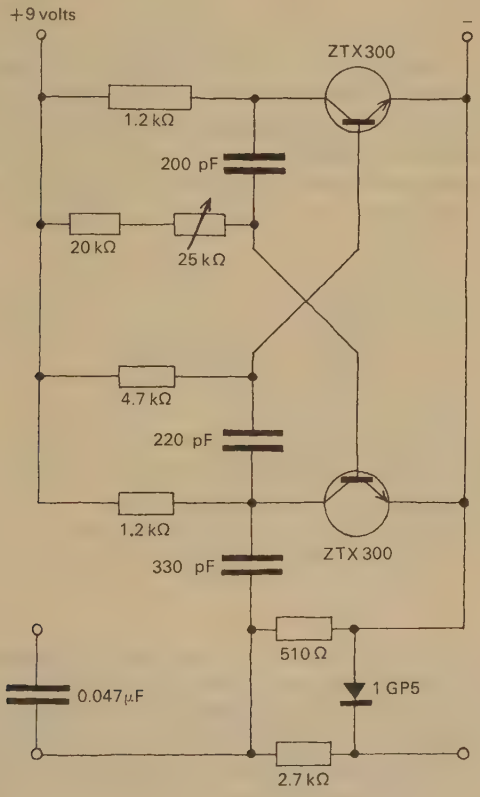


Figure 54

The klystron power supply may produce a 'smooth' reflector voltage which still has a small mains ripple on it. The 200 kHz pulse generator shown in figure 54 has a capacitor (0.047 μF) which may be connected between the power supply's earth and variable negative output, to reduce ripple (figure 55).

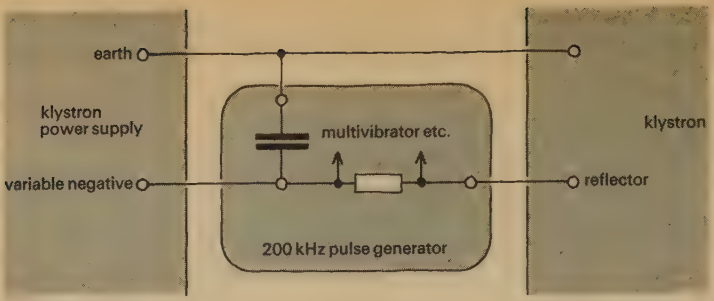


Figure 55

Not all klystrons commercially available can have the connections to their power supplies broken in the convenient way shown in figure 55. Sometimes there may be a pair of sockets, one going to the reflector supply and the other to the reflector, normally bridged by a connector which can be removed. Sometimes it might be necessary to open the power supply and adapt it. When so adapted the klystron is potentially a radio transmitter. Post Office regulations prohibit the conveyance of messages by such a microwave link, and suppliers of equipment do not encourage their customers to break the law by explaining in detail how it is to be done.

Path of the microwaves

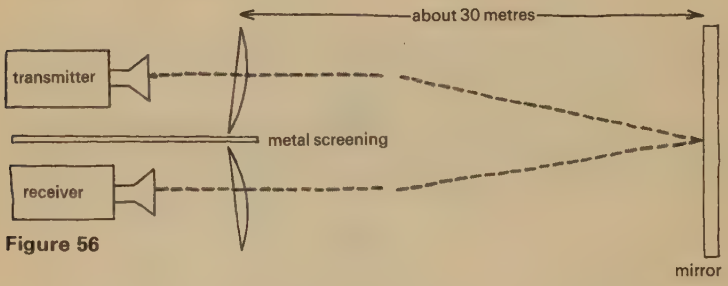


Figure 56

A wax lens should be used to produce a strong beam of microwaves directed towards the mirror (figure 56). A second wax lens needs to be used to concentrate the returning microwaves on the receiver. There is a danger of radiation travelling direct from the transmitter to the receiver or being scattered into the receiver by one or other lens, so that a metal screen between them, and a distance of about a metre between the centres of the lenses, are advisable.

The mirror can be a metal sheet, metal foil pinned on a board, or metallized plastic stretched on a frame. Variation of more than a few millimetres from flatness reduces performance. The area should be more than half a square metre. It will help in setting up the path if the mirror reflects light, otherwise it will be necessary to

transmit audio-modulated microwaves and use an amplifier and loudspeaker with the receiver while adjusting the lenses and mirror to the best alignment possible.

When the mirror's best angle is found, it is wise to check that tilting it slightly will eliminate the receiver's signal, otherwise, unless the apparatus is in the middle of a large open space, it is possible that stray reflections may confuse the result.

The receiver

The output from the receiver is weak and needs amplifying with a pre-amplifier before being fed into the oscilloscope. In some cases it will be necessary to screen all connections between the receiver and pre-amplifier, lest they pick up radio signals or the pulses from the pulse generator. This screening may be a tin box, independently earthed, in which the receiver, amplifier, and its battery are enclosed, there being a small opening for the receiver's horn. Parts of the receiver may also need earthing. A coaxial connection may be needed between the pre-amplifier and the oscilloscope, depending on the pre-amplifier's output impedance and the amount of stray signal to be picked up.

The oscilloscope

A double-beam oscilloscope is needed, with the timebase set to 1 cm μs^{-1}. The pulse fed to the klystron reflector should also be fed to one input, preferably the one giving the lower trace, set at, say, 2 V cm^{-1}, and the timebase should be triggered from this input. A wire can go straight from the reflector to the oscilloscope input if this is set for a.c. The output from the pre-amplifier goes to the other oscilloscope input (upper trace) set to 0.1 V cm^{-1}. When the klystron power supply's reflector voltage output is set to a suitable value, which must be found by adjustment, the upper trace should display the returning microwave pulse, which starts slightly to the right of the pulse on the lower trace (figure 57 *b*). There may be difficulty keeping the microwave pulse constant because the klystron has not been switched on long enough or because fluctuations in mains voltage alter the reflector voltage slightly.

Measurements

The slight delay between the transmitting pulse and the returning pulse might be due to other causes than the time the microwaves are travelling. It is therefore necessary to take two metal screens and use one to block off all microwaves from the transmitter while using the other to send enough of them into the receiver to give the same sized trace as before (figure 57 *a*). The shift of the receiver trace relative to the transmitter trace, as these two screens are removed (figure 57 *b*) is then a true measure of the time taken for the microwaves to travel to the distant mirror and back again. Figure 58 shows the two oscilloscope displays enlarged.

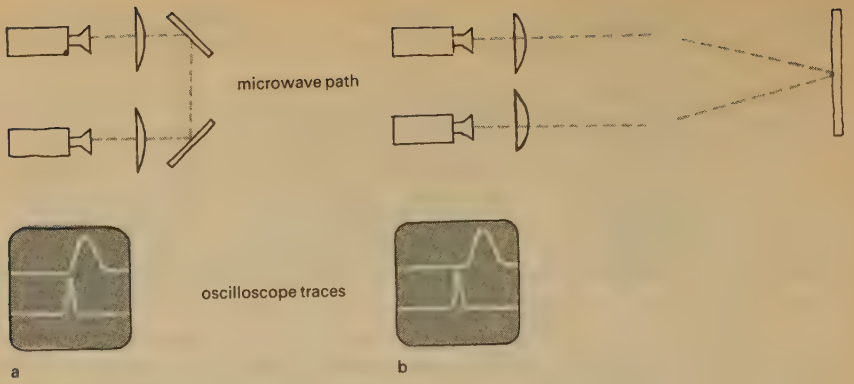


Figure 57

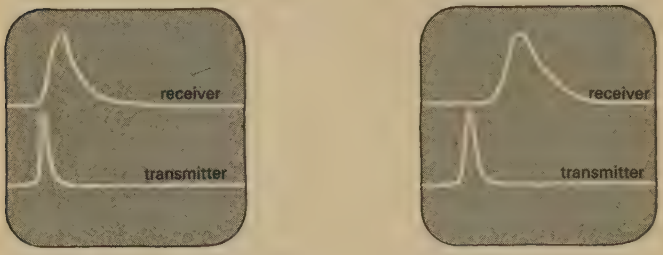


Figure 58

Problems

The problems of setting up this demonstration vary from school to school, particularly in the following ways.

1 The apparatus will occupy a lot of table space somewhere which may be distant from the physics laboratories and from any source of electrical power.

2 The pulse generator, klystron, receiver, and amplifier may not be an effective combination. None of them could easily be specified, or easily checked against a specification.

3 Mains voltage may fluctuate particularly widely, making it difficult to keep the klystron properly pulsed.

4 Local radio transmitters may be powerful enough to induce confusing pick-up on the receiver leads.

5 Some fixed large area of conducting material may be available for the mirror which will enable a longer path to be used without loss of signal strength. The intensity of the returning microwaves will, in some circumstances, vary with the square of the mirror's area.

Appendix C
Astronomical evidence of the speeds of light and of radio waves

The letter to *Nature* reproduced below reports evidence indicating that light and radio waves from stars travel to the Earth at the same speed. Certain 'flare stars' produce fluctuations in optical and radio emission, and the evidence concerns the fact that these fluctuations arrive at the Earth very close in time; there is less than a few minutes' difference between their arrival times in a total travel time of about 10^6 minutes. Were there any small difference in the speeds of the radio and light radiation, the large distance the radiation travels would produce a rather large difference in the time of arrival. The observations show that the speeds do not differ by more than 1 in 10^6. Slide 4.1 presents a simplified version of the data in the letter.

'Radio Astronomy

Relative velocity of light and radio waves in Space

'Since the velocity of light is one of the most fundamental physical quantities, any measures which indicate its constancy with wavelength are of basic physical interest. Experimental determinations in the optical region and in the short wave region indicate a constancy to 1 part in 10^6. Astronomical measurements, because of the great distances involved, hold the promise of extremely precise determinations of the relative velocities of electromagnetic waves from the observation of distant events that occur simultaneously or nearly simultaneously at different wavelengths. The recent observations of optical and radio events in flare stars now provide the basis for such determinations.

Star	Parallax	Light time /10^6 minutes	Time difference radio-optical /minutes	Radio-frequency /MHz	Type of burst	Date
UV Ceti (*L* 726-8*AB*)	0.375″	4.6	−2	240	1	Mean of 23 events
			+2	408	2	25 Oct. 1963
*V*371 *Orionis* (Wachmann's star)	0.066″	26.0	+3 phot. +11 vis.	410	2?	30 Nov. 1962
YZ C Mi (Ross 882)	0.151″	11.0	−1	240	1	18 Feb. 1961
EV Lac. (+43°4305)	0.198″	8.7	+2	240	1?	7 Aug. 1961

Table [2]

[6]Table [2] presents the relevant data now available concerning optical and radio events in four flare stars (column 1). The second and third columns contain the stellar parallax and the corresponding time of light travel. Column 4 gives the interval of time in minutes between the first observation of the optical event and the beginning of the radio event. The photographic observations were made at intervals of 2 minutes with

exposures of several seconds. The successive columns contain the radio frequency of observation, the type of stellar burst, and the date of observation.

'As yet we have no reliable theory to establish the temporal sequence of events in flare stars. The Type 2 events appear to be most analogous to the Type II radio bursts on the Sun, in that the beginning of the higher-frequency burst follows the beginning of the observed flare and the low-frequency burst occurs later. This is generally explained as arising from the decrease with height of critical frequency in the solar atmosphere and the progress outward of the disturbance from the solar surface. The phenomenon of October 25, 1963, on *UV Ceti*, however, occurs approximately 5 times faster than that on the Sun, where the delay between the optical and radio emission is typically 10 minutes. It is, therefore, reasonable to conclude that the observed 2-minute time difference between the optical and radio events on *UV Ceti* arise in the stellar atmosphere. On this assumption, since the optical flares may have begun as much as 2 minutes earlier than observed, it may be concluded that the velocity of light and radio waves is constant to 4 parts in 10^7. From Lorentz's equation for the velocity of radiation in an ionized medium, the corresponding upper limit to the electron density in free space between the Earth and *UV Ceti* would be 570 electrons cm^{-3}. For the solar neighbourhood in the galaxy one would not expect the electron density to exceed roughly 0.1 electron cm^{-3}.

'The Type 1 flare-star radio bursts, however, are not clearly identified with a type of solar radio burst. Nevertheless, it is difficult to believe that the radio burst, if associated with an optical flare, should or could precede the flare. Since the negative time differences in Table [2] include an uncertainty of 2 minutes in the time between exposures, the negative values are not necessarily significant. This again suggests a possible coincidence within the errors of observation between the optical and the radio events. The observations, however, leave open the possibility that the optical radiation travels slightly more slowly than the radio radiation if the two velocities are not, indeed, identical.

'Allowing for a maximum uncertainty of 5 minutes in the temporal sequence for flare stars, we can conclude with confidence that the velocity of light and radio waves is the same to within one part in a million over a range in wavelength of 2 million (0.54 $\mu m - 1.2$ m) and, with somewhat less confidence, to 4 parts in 10^7. The product $(c/\Delta c)$ (λ_2/λ_1) is thus 2 to 5×10^{12}. Terrestrial measurements give a similar product. Thus the observational evidence still supports the concept that the velocity of electromagnetic radiation in space is independent of wavelength.

'We thank Dr John Findlay, who suggested that these comparisons might be of general interest.

Bernard Lovell
University of Manchester,
Nuffield Radio Astronomy Laboratories,
Jodrell Bank,

Fred L. Whipple
Leonard H. Solomon
Smithsonian Astrophysical Observatory,
Cambridge, Massachusetts.'

(*Reproduced from* Nature, *4930, page 377, 1964, with permission.*)

Appendix D
Applications

The following are samples of the kind of information that it is hoped teachers will collect to illustrate the uses of the ideas presented in this book.

1 The effects of vibration on man

We know that the slow oscillations of a rolling ship can produce sea-sickness, although the origin of car-sickness is less clear. Machine operators are subject to more rapid vibrations; the pneumatic road drill being an extreme example. Some effects of oscillations of various frequencies are shown in figure 59.

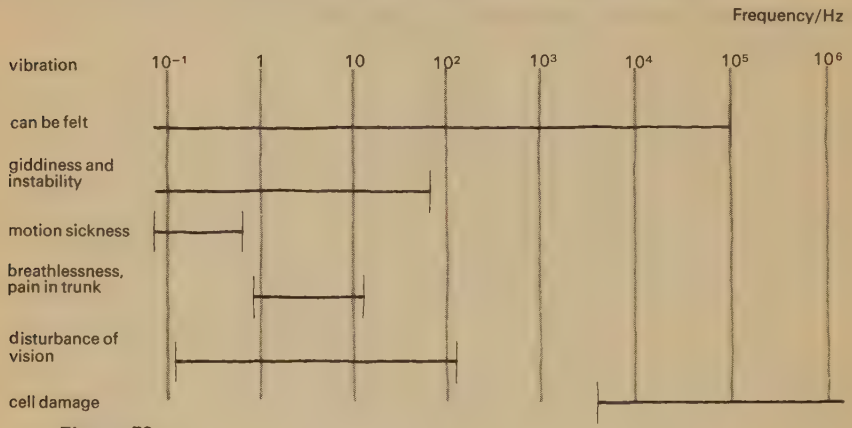


Figure 59

Most serious effects are due to *resonance* – when the natural frequency of oscillation of some part of the body is the same as the frequency with which it is driven. This has been studied by sitting a person on a vibrating platform. Figure 60 shows the motion of the abdomen wall at various frequencies.

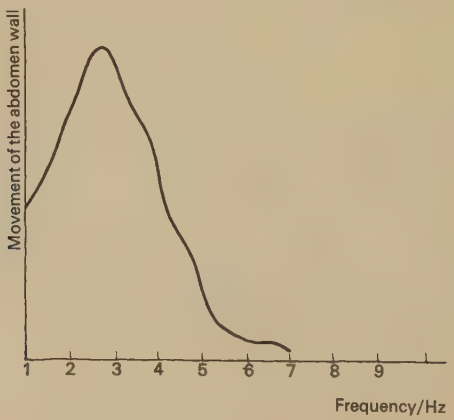


Figure 60

The tolerance of vibration by human beings varies with frequency (figure 61). Such studies are of especial importance in designing aircraft and space probes.

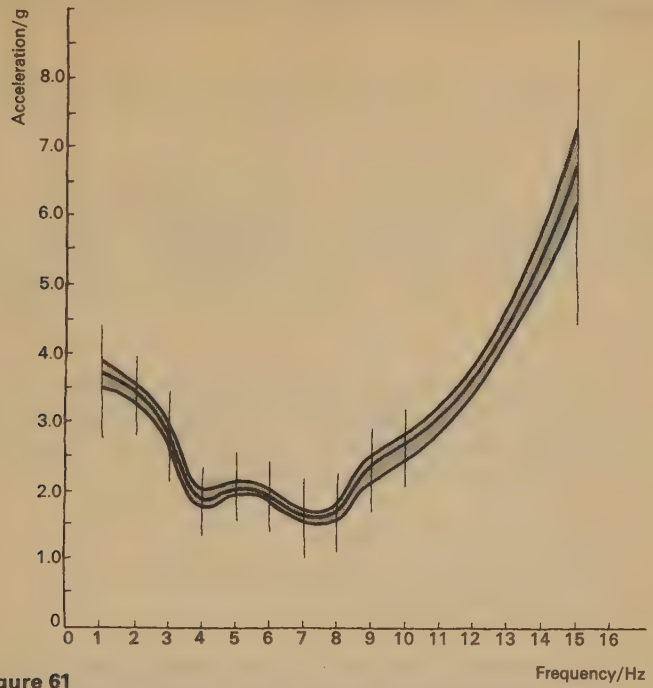


Figure 61

Human vibration tolerance. The curves show the value, and the range, of the limit of tolerable acceleration at various frequencies.
From Magid, E. B., Coermann, R. R., Ziegenruecker, G. H. (1960) Aerospace medicine, **31**, *page 915.*

Human vibration engineering is also important in designing hand-operated machine tools. The use of such a tool for intricate work would be very difficult if it vibrated at a resonant frequency of the hand–arm system.

2 Spectroscopy

If one could see the atoms in a molecule vibrating, and time their oscillations, one could obtain useful information about the stiffness k of the bonds between them, using $2\pi f = \sqrt{k/m}$. Although the vibrating atoms cannot be seen, the frequency at which they absorb radiation can be found. Spectroscopy is thus a valuable tool for studying the vibrations of electrons, atoms, molecules, or ions.

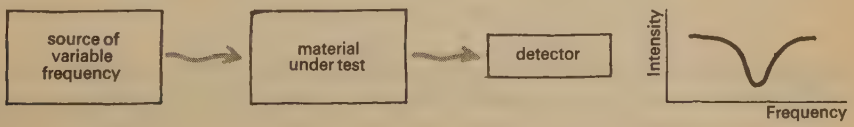


Figure 62

At its simplest, a source of variable frequency sends radiation to a detector through the material under test. Such methods are appropriate if the frequency of vibration is relatively slow, so that the wavelength of the electromagnetic radiation is more than a few millimetres.

Many interesting vibrations are faster, but sources of infra-red or visible light of variable frequency are not available.

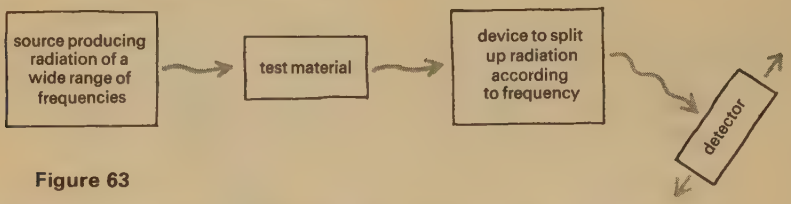


Figure 63

Radiation of a wide range of frequencies is then shone on the test material, and a device (a grating or prism, for example) sends the radiation of each frequency off in a different direction. Alternatively, the detector could in principle be tuned to each frequency in succession, although this is less useful in practice.

Uses of spectroscopic information

The stiffness of bonds in molecules, or in solids, may be found. For example, the bond stiffness of H_2 is 5.2×10^2 N m^{-1}.

The analysis of complex organic compounds is assisted by studying their infra-red absorption spectra, for many types of bond tend to absorb at much the same frequency even though the atoms form part of different molecules. The spectrum can then be used as a means of indicating which bonds are present. For instance, aliphatic C—C bonds oscillate in an in-and-out (stretching) manner at a little below 10^{14} Hz.

Dyes, whose function is to be coloured, must absorb visible radiation strongly at selected frequencies. It is possible to design molecules which will absorb at a desired frequency.

At microwave frequencies, the spinning motion of molecules can be studied, and information about the length of bonds and the masses of the atoms obtained.

3 Ultrasonic flaw detection

Ultrasonic flaw detection is an example of a non-destructive testing method. Such methods are used where the material to be tested must not be cut up, broken down chemically, or even removed from its working position.

Ultrasonic flaw detectors work on a simple pulse-echo principle. A probe which produces ultrasonic pulses in the frequency range from 0.5 to 12 MHz is placed in contact with the material to be inspected. The pulses travel through the material in

straight lines and are reflected from its rear surface, returning to the probe which picks them up and so acts also as a detector. If, however, there is a crack or flaw in the material, the pulses will also be reflected back from it and will give a signal in the probe before the signal due to reflection from the back of the material.

The signals are displayed on an oscilloscope screen and the position of the flaw and its approximate size can be judged from the position and height of the flaw echo.

One important application of the method is the examination of railway track, particularly near the ends of rail sections which are joined together by fishplates. These receive a severe pounding, audible as the familiar clickety-clack of the wheels, and a fracture can be disastrous.

Lists of slides, films, loops, books, and apparatus

Slides

Slide 4.1
A simplified version of the table given in Appendix C. It shows that the time difference between radio and light emissions from certain stars is small.

Slide 4.2
Reproduced with permission from PSSC Physics. Shows pulses crossing on a rope.

Slide 4.3
Reproduced with permission from PSSC Physics. Shows pulses crossing on a spring.

Slide 4.4
Infra-red absorption due to a thin layer of sodium chloride.

Slides 4.5(1) to 4.5(7) and 4.5(9) all concern Decca Navigation. The Decca Navigator Company Limited have kindly provided the material for these slides.

Slide 4.5(1)
Hyperbolic pattern of constant phase difference lines produced by Master and Red slave transmitters.

Slide 4.5(2)
Overlapping hyperbolae due to Master and Red slave and Master and Green slave.

Slide 4.5(3)
Complete pattern due to Master station and Red, Green, and Purple slaves superimposed on outline of S.E. England.

Slide 4.5(4)
Two Decometers indicating a 'fix'. (Lane identification meter also shown.)

Slide 4.5(5)
Part of an actual map showing Decca lanes.

Slide 4.5(6)
Chart showing frequencies, wavelengths, and lane widths.

Slide 4.5(7)
Apparatus used in aircraft.

Slide 4.5(8)
Photograph, British European Airways. Display unit installed in BAC Super One Eleven cockpit.

Slide 4.5(9)
Block diagram of receiver elements.

Teachers may find the following notes helpful if they use the Decca Navigation slides.

Notes on slides 4.5(1) to 4.5(9)

Decca Navigation

The system of navigation illustrated by these slides is based on interference of waves from two sources. In the ripple tank a pattern of hyperbolae shows lines along which the waves from two dippers are exactly out of step or out of phase. The Decca Navigation system uses an analogous arrangement to produce a set of 'lines' along each of which the radio waves from two transmitters arrive exactly out of step. Of course the lines are not visible but they would be detected by the fact that a radio receiver tuned to the waves would indicate a minimum signal. Slide 4.5(1) shows the zero signal paths for waves from two radio stations labelled Master and Red. In the ripple tank the two dippers were in phase because they were attached to a rigid horizontal beam. A steady pattern would of course arise provided the two dippers kept at a constant phase difference. In the Decca Navigation system the phase difference of the waves starting from the two transmitters is kept constant by using a Master transmitter to send out waves and a slave transmitter synchronized with it.

A pilot in an aircraft equipped with a suitably tuned receiver could fly along any of the zero phase difference lines simply by steering so as to keep the resultant signal zero. But even if he knew which line he was on at the start of his flight he still could not fix his position. To do this he needs another set of hyperbolae intersecting the first set. This is shown in slide 4.5(2). The same Master transmitter as before has another slave station labelled Green. A different wavelength is used and another set of zero phase difference hyperbolae (shown in dashes) exists. If the pilot has another receiver tuned to this second frequency it will indicate zero signal when his aircraft is on one of the second lot of hyperbolae. When both receivers indicate zero signal he knows he is at an intersection of the two sets of hyperbolae. But as there are several such intersections he must at the beginning of his flight know just where he is. To give better coverage a third set of hyperbolae is added, shown dotted in slide 4.5(3) and produced by Master and Purple slave. This slide also shows the map of S.E. England with respect to this particular Decca chain of stations. Two Decca 'lanes' are also shaded in and labelled C and G respectively – C is a Green lane and G a Red lane. On the base lines between master and slaves the widths of the respective lanes are

Red 440 m **Green** 587 m **Purple** 352 m.

Thus if the pilot had to depend only on zero signal times he would only be able to check his position at line intersections – every 440 m, or 587 m, or 352 m for this Decca chain. Actually, the aircraft (or ship) carries receivers which feed their outputs, say from master and red slave, into a device called a Decometer which indicates the phase difference between the two signals. Slide 4.5(4) shows how the system is used to give the position for the point indicated by the arrow. In the rectangle a part of the region where C and G lanes overlap is shown; by looking at

this chart one can estimate the point position as being between lines 16 and 17 in the Red 'zone' and between 35 and 36 in the Green 'zone'. Actually the pilot would only read the two Decometer dials shown below the rectangle; the lefthand dial tells him he is at 16.28 in G lane 16–17 and the righthand dial tells him he is at 35.80 in the C lane 35–36. (The middle dial is for lane identification and enables him to check which lane he is in – as shown, the pointer in the quadrant confirms that he is in red lane of whole number 16.) Having in this way found a pair of co-ordinates for his position he can read this off on a map which has the Decca lanes drawn on it as shown in slide 4.5(5).

Many students might go from slide 4.5(5) to slide 4.5(7) which shows the items of the equipment which would be used in a modern aircraft; a computer automatically plots the aircraft position on a display unit. Slide 4.5(8) shows the display unit in the cockpit of the aircraft.

A few students might appreciate the elegance of the Decca system after seeing slide 4.5(6) and discussing the information shown. The basic frequency f is 14.16 kHz. The Master transmitter does not radiate on frequency f but on its harmonic $6f$. The Red, Green, and Purple slave transmitters radiate at frequencies $8f$, $9f$, and $5f$ respectively; consider the Master ($6f$) and Red ($8f$) frequencies. The receiver in the aircraft multiplies the former frequency by 4 and the latter by 3, thus converting them both to $24f$. Having done this the Decometer indicates the phase difference between the two sets of waves *as though they had been radiated at a frequency of* $24f$. This corresponds to a wavelength of approximately 880 m. With this information and some thought students will probably be able to work out the red lane baseline width as 440 m.

Slide 4.5(9) shows as a block diagram the receiver elements carried in the aircraft. The signals at frequencies $5f$, $6f$, $9f$, and $8f$ are first amplified and then fed into the multipliers where they are distorted to increase the harmonic content, after which suitably tuned circuits select the required harmonics. Discriminators then compare the phases of master station and slave station signals, the phase difference for each pair being indicated on the appropriate Decometer.

A demonstration using a piano might help students to appreciate the multiplication technique:

> Strike middle C and use a microphone coupled to a CRO in order to show that the note is not pure. Now hold the top C key down, strike middle C, then apply the 'soft' pedal when the first harmonic of middle C will be heard, having been selected from the impure note, by the tuned C string.

Wireless world, August 1969, has an article by F. S. Stringer of R.A.E. on 'Hyperbolic radio navigation systems' (pages 353–7).

The proceedings of the Institution of Electrical Engineers, Vol. 105, Part B Supplement No. 9, 1958, has an article 'The Decca Navigator System for ship and aircraft use' by C. Powell of Decca (pages 225–34).

Films and film loops

Page numbers of references in this *Guide* appear in bold type.

Film

'The velocity of gamma rays.' 16 mm, 16 minutes, colour, sound. No. 21.7853, on hire from the Rank Film Library, Rank Audio Visual Ltd., PO Box 70, Great West Road, Brentford, Middlesex. **26.**

Film loops

Probably useful are:

'Tacoma Narrows Bridge collapse.' Ealing Scientific: Nos. A80-2181/1 (super 8), A80-2181/2 (standard 8). **64.**

'Wind-induced oscillations.' Penguin, No. XX1671. **64, 112.**

'Wind-induced oscillations' was made in conjunction with the Advanced Physics Project.

Optional:

'Measurement of *"G"*.' Ealing Scientific: Nos. A80-2124/1 (super 8), A80-2124/2 (standard 8). **68.**

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Footnote

* *Science journal* reprints are no longer available. These articles will, however, appear in a collection of these reprints entitled *Physics and the engineer*, to be published in 1972 as part of the Nuffield Advanced Physics publications (Penguin).

For teachers

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A *Teachers' guide* has been produced for each of the ten Units forming the Advanced Physics course. This is the *Guide* for Unit 4, *Waves and oscillations*. It is intended to provide whatever information and ideas are required for the day-to-day teaching of the Unit. The book begins with an Introduction setting out the purpose of the Unit, a summary of the Unit, and a list of suggested experiments. Following this, the main text consists of three Parts, 'Waves of many sorts', 'Mechanical waves', and 'Mechanical oscillations'. It contains teaching suggestions, details of experiments, and a commentary giving background information and other guidance. There are also Appendices on 'Frequencies of transmission', 'Measuring the speed of microwaves', 'Astronomical evidence of the speeds of light and of radio waves', and 'Applications', and lists of slides, films, books, and apparatus.

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